

Find the equation of the derivative for each of the following functions. Also, find the instantaneous rate of change for the function when $x = 4$ and $x = -1$.

a) $f(x) = 3x - 8$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{3(x+h) - 8 - (3x - 8)}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{3x + 3h - 8 - 3x + 8}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{3h}{h}$$

$$f'(x) = 3$$

$$f'(4) = 3$$

$$f'(-1) = 3$$

b) $y = 20x + x^2$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{20(x+h) + (x+h)^2 - (20x + x^2)}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{20x + 20h + x^2 + 2xh + h^2 - 20x - x^2}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{20h + 2xh + h^2}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{h(20 + 2x + h)}{h}$$

$$f'(x) = 20 + 2x + 0$$

$$f'(x) = 2x + 20$$

$$f'(4) = 2(4) + 20$$

$$f'(4) = 28$$

$$f'(-1) = 2(-1) + 20$$

$$f'(-1) = 18$$

c) $y = 2x^3 + 4$

$$f'(x) = \lim_{h \rightarrow 0} \frac{2(x+h)^3 + 4 - (2x^3 + 4)}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{2(x+h)(x^2 + 2xh + h^2) + 4 - 2x^3 - 4}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{2(x^3 + 2x^2h + xh^2 + x^2h + 2xh^2 + h^3) - 2x^3}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{2(x^3 + 3x^2h + 3xh^2 + h^3) - 2x^3}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{2x^3 + 6x^2h + 6xh^2 + 2h^3 - 2x^3}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{h(6x^2 + 6xh + 2h^2)}{h}$$

$$f'(x) = 6x^2 + 6x(0) + 2(0)^2$$

$$f'(x) = 6x^2$$

$$f'(4) = 6(4)^2$$

$$= 96$$

$$f'(-1) = 6(-1)^2$$

$$= 6$$

d) $f(x) = x^2 - 9x + 17$

$$f'(x) = \lim_{h \rightarrow 0} \frac{(x+h)^2 - 9(x+h) + 17 - (x^2 - 9x + 17)}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 - 9x - 9h + 17 - x^2 + 9x - 17}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{2xh + h^2 - 9h}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{h(2x + h - 9)}{h}$$

$$f'(x) = 2x + 0 - 9$$

$$f'(x) = 2x - 9$$

$$f'(4) = 2(4) - 9$$

$$= -1$$

$$f'(-1) = 2(-1) - 9$$

$$= -11$$

$$e) f(x) = \frac{x(x+1)}{2}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{\frac{(x+h)(x+h+1)}{2} - \frac{x(x+1)}{2}}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{x^2 + xh + x + xh + h^2 + h - x^2 - x}{2h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{2xh + h^2 + h}{2h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{h(2x+h+1)}{2h}$$

$$f'(x) = \frac{2x+0+1}{2}$$

$$f'(x) = \frac{2x+1}{2}$$

$$f'(x) = x + \frac{1}{2}$$

$$f'(4) = 4 + \frac{1}{2} = \frac{9}{2}$$

$$f'(-1) = -1 + \frac{1}{2} = -\frac{1}{2}$$

$$f) f(x) = \frac{1}{x}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{\frac{x - (x+h)}{x(x+h)}}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{x - x - h}{h(x)(x+h)}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{-1h}{h(x)(x+h)}$$

$$f'(x) = \frac{-1}{x(x+0)}$$

$$f'(x) = \frac{-1}{x^2}$$

2) State whether the functions are increasing, decreasing, or neither when $x = 4$ for each function in #1. How do you know?

a, b, c, e are increasing since $f'(4)$ is positive

d, f are decreasing since $f'(4)$ is negative

3)a) State the derivative of $f(x) = x^3$

$$f'(x) = \lim_{h \rightarrow 0} \frac{(x+h)^3 - x^3}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{(x+h)(x^2 + 2xh + h^2) - x^3}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{x^3 + 2x^2h + xh^2 + x^2h + 2xh^2 + h^3 - x^3}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{3x^2h + 3xh^2 + h^3}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{h(3x^2 + 3xh + h^2)}{h}$$

$$f'(x) = 3x^2 + 3x(0) + (0)^2$$

$$f'(x) = 3x^2$$

b) Evaluate $f'(-6) = 3(-6)^2$
 $= 108$

c) Determine the equation of the tangent line at $x = 6$

$f(6) = (6)^3$
 $= 216$ ∴ point $(6, 216)$ is on the tangent line

$f'(6) = 3(6)^2$
 $= 108$ ∴ slope of the tangent line is 108

$y = mx + b$

$216 = 108(6) + b$

$b = -432$

$y = 108x - 432$

Answer Key

1)a) $f'(x) = 3, f'(4) = 3, f'(-1) = 3$ b) $f'(x) = 20 + 2x, f'(4) = 28, f'(-1) = 18$

c) $f'(x) = 6x^2, f'(4) = 96, f'(-1) = 6$ d) $f'(x) = 2x - 9, f'(4) = -1, f'(-1) = -11$

e) $f'(x) = x + \frac{1}{2}, f'(4) = \frac{9}{2}, f'(-1) = -\frac{1}{2}$ f) $f'(x) = -\frac{1}{x^2}, f'(4) = -\frac{1}{16}, f'(-1) = -1$

2) a,b,c and e are increasing functions when $x = 4$ since the instantaneous rate of change is positive

and f are decreasing when $x = 4$

3)a) $f'(x) = 3x^2$ b) 108 c) $y = 108x - 432$