W4 – Equations of Planes in Scalar Form Unit 6

MCV4U

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SOLUTIONS

1) Write the scalar equation of each plane given the normal $\vec{n}$ and a point P on the plane.

a) $\vec{n} = [1, -3, -4]$, $P(-2, 5, 7)$

$1(-2) - 3(5) - 4(7) + D = 0$

$D = 45$

$x - 3y - 4z + 45 = 0$

b) $\vec{n} = [0, -3, 5]$, $P(8, -7, 3)$

2) Write a scalar equation of each plane, given its vector or parametric equation.

a) $[x\ y\ z] = [7, -3, 4] + s[1, -2, -3] + t[4, -3, 1]$

$\vec{s} \times \vec{t} = [-2(1) - (-3)(-3), -3(4) - 1(1), 1(-3) - (-2)(4)]$

-2 -3
-3 1
1 4
-2 -3

$= [-11, -13, 5]$

$\vec{n} = [-11, -13, 5]$

$-11(7) - 13(-3) + 5(4) + D = 0$

$D = 18$

$-11x - 13y + 5z + 18 = 0$

b) $\pi_1 \begin{cases} x = 4 + t + 2s \\ y = 3 - 2t - 3s \\ z = -1 + 3t + s \end{cases}$

$\vec{m}_1 = [1, -2, 3]$ $\vec{m}_2 = [2, -3, 1]$

$\vec{n} = \vec{m}_1 \times \vec{m}_2 = [-2(1) - 3(-3), 3(2) - 1(1), 1(-3) - (-2)(2)]$

-2 -3
3 1
1 2
-2 -3

$= [7, 5, 1]$

$\vec{n} = [7, 5, 1]$

$7(4) + 5(3) + 1(-1) + D = 0$

$D = -42$

$7x + 5y + z - 42 = 0$

3) For each situation, write the [redacted] scalar equation[redacted] of the plane.

a) has normal $\vec{n} = (7, 9, -1)$ and includes the point $(3, -2, 4)$

$7x + 9y - z + D = 0$

$7(3) + 9(-2) - 1(4) + D = 0$

$D = 1$

$7x + 9y - z + 1 = 0$

b) contains direction vectors $\vec{a} = [-1, 2, 8]$ and $\vec{b} = [2, -1, 3]$ and includes the point $(2, -7, 8)$

$\vec{n} = \vec{a} \times \vec{b} = [2(3) - 8(-1), 8(2) - (-1)(3), -1(-1) - 2(2)]$

2 -1
8 3
-1 2
2 -1

$= [14, 19, -3]$

$14x + 19y - 3z + D = 0$

$14(2) + 19(-7) - 3(8) + D = 0$

$D = 129$

$14x + 19y - 3z + 129 = 0$

c) parallel to the xz-plane and includes the point (7, 8, −1)

$\boxed{y=8}$

Long way:

$\vec{m}_1 = [1,0,0]$

$\vec{m}_2 = [0,0,1]$

$\vec{n} = \vec{m}_1 \times \vec{m}_2 = [0,-1,0]$

$$\begin{array}{cc} 0 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 0 \end{array}$$

$0x - 1y + 0z + D = 0$

$0(7) - 1(8) + 0(-1) + D = 0$

$D = 8$

$\boxed{y=8}$

d) contains the points (3, 8, −1), (−8, 9, −4), and (1, −3, 2)

$\vec{m}_1 = [-11,1,-3]$

$\vec{m}_2 = [-2,-11,3]$

$\vec{n} = \vec{m}_1 \times \vec{m}_2 = [-30,39,123]$

$$\begin{array}{cc} 1 & -11 \\ -3 & 3 \\ -11 & -2 \\ 1 & -11 \end{array}$$

$-30x + 39y + 123z + D = 0$

$-30(3) + 39(8) + 123(-1) + D = 0$

$D = -99$

$\boxed{-30x + 39y + 123z - 99 = 0}$

e) contains the line $[x, y, z] = [4, -3, -2] + s[3, -2, 1]$ and parallel to the line defined by the parametric equations $\ell \begin{cases} x = 5 + 3s \\ y = 1 - s \\ z = 2 + 4s \end{cases}$

$P_0: (4,-3,-2)$

$\vec{m}_1 = [3,-2,1]$

$\vec{m}_2 = [3,-1,4]$

$\vec{n} = \vec{m}_1 \times \vec{m}_2 = [-2(4)-1(-1),\ 1(3)-3(4),\ 3(-1)-(-2)(3)]$

$= [-7,-9,3]$

$$\begin{array}{cc} -2 & -1 \\ 1 & 4 \\ 3 & 3 \\ -2 & -1 \end{array}$$

$-7x - 9y + 3z + D = 0$

$-7(4) - 9(-3) + 3(-2) + D = 0$

$D = 7$

$\boxed{-7x - 9y + 3z + 7 = 0}$

f) contains the point (2, −1, 8) and perpendicular to the line $[x, y, z] = [1, -2, -3] + s[5, -4, 7]$

$\vec{n} = [5,-4,7]$

$5x - 4y + 7z + D = 0$

$5(2) - 4(-1) + 7(8) + D = 0$

$D = -70$

$\boxed{5x - 4y + 7z - 70 = 0}$

g) parallel to the plane $-3x + 2y + 5z + 8 = 0$ and includes the point $(5,-7,8)$

$\vec{n} = [-3,2,5]$

$-3x + 2y + 5z + D = 0$

$-3(5) + 2(-7) + 5(8) + D = 0$

$D = -11$

$$\boxed{-3x + 2y + 5z - 11 = 0}$$

h) contains the lines $[x,y,z] = [4,-1,0] + s[-2,1,3]$ and $[x,y,z] = [-2,4,3] + s[-6,5,7]$

$\vec{m_1} = [-2,1,3]$

$\vec{m_2} = [-6,5,7]$

$\vec{n} = \vec{m_1} \times \vec{m_2} = [-8,-4,-4]$

$= -4[2,1,1]$

1	5
3	7
-2	-6
1	5

$2x + y + z + D = 0$

$2(4) + (-1) + 0 + D = 0$

$D = -7$

$2x + y + z - 7 = 0$

Determine the angle between the planes $x + 2y - 3z - 4 = 0$ and $x + 2y - 1 = 0$. *(Hint: the angle between 2 intersecting planes should be equal to the angle between their normals)*

$\vec{n_1} = [1,2,-3]$

$n_2 = [1,2,0]$

$\cos\theta = \dfrac{\vec{n_1} \cdot \vec{n_2}}{|\vec{n_1}||\vec{n_2}|}$

$\cos\theta = \dfrac{5}{(\sqrt{14})(\sqrt{5})}$

$\theta \simeq 53.3°$

5) Write the vector equation and parametric equations of the plane: $3x - y + 2\cancel{z} - 4 = 0$

x-int: $3x = 4$

$x = \frac{4}{3}$ $(\frac{4}{3},0,0)$

y-int: $-y = 4$

$y = -4$ $(0,-4,0)$

z int: $2z = 4$

$z = 2$ $(0,0,2)$

$\vec{XY} = [-\frac{4}{3},-4,0]$

$= 3\vec{XY} = [-4,-12,0]$

$-\frac{1}{4}(3\vec{XY}) = [1,3,0]$

$\vec{m_1} = [1,3,0]$

$\vec{YZ} = [0,4,2] = 2[0,2,1]$

$\vec{m_2} = [0,2,1]$

$[x,y,z] = [0,0,2] + t[1,3,0] + s[0,2,1]$

$$\begin{cases} x = t \\ y = 3t + 2s \\ z = 2 + s \end{cases}$$

6) Find a vector that is normal and one that is parallel to each plane.

a) $x + 2y + 2z - 5 = 0$

normal: $\vec{n} = [1,2,2]$

parallel: $\vec{m} = [-2,1,0]$

b) $5x + 2z = 7$

normal: $\vec{n} = [5,0,2]$

parallel: $\vec{m} = [-2,0,5]$

c) $5y = 8$

perpendicular: $\vec{n} = [0,5,0]$

parallel: $\vec{m} = [2,0,7]$

anything in xz-plane.

7) Consider the plane $-x + 4y + 2z + 6 = 0$

a) Determine a normal vector, $\vec{n}$, to the plane.

$\vec{n} = [-1,4,2]$

b) Determine the coordinates of two points, S and T, on the plane.

$-x+4(0)+2(0)+6=0$

$x=6$

$S(6,0,0)$

$-0+4(0)+2z+6=0$

$z=-3$

$T(0,0,-3)$

c) Determine $\overrightarrow{ST}$

$\overrightarrow{ST} = [0-6, 0-0, -3-0]$

$= [-6,0,-3]$

d) Show that $\overrightarrow{ST}$ is perpendicular to $\vec{n}$.

$\overrightarrow{ST} \cdot \vec{n} = [-6,0,-3] \cdot [-1,4,2]$

$= 6 + 0 - 6$

$= 0$

$\therefore$ perpendicular.

8) State the scalar equation of each of the following

a) the xy-plane

$z=0$

b) the xz-plane

$y=0$

c) the yz-plane

$x=0$

ANSWER KEY:

1. a) $x-3y-4z+45=0$ **b)** $-3y+5z-36=0$ **2. a)** $-11x-13y+5z-18=0$ **b)** $7x+5y+z-42=0$

3. Answers may vary for some vector equations. **a)** $7x+9y-z+1=0$, $[x,y,z]=[3,-2,4]+t[-9,7,0]+s[0,1,9]$

b) $14x+19y-3z+129=0$, $[x,y,z]=[2,-7,8]+t[-1,2,8]+s[2,-1,3]$ **c)** $y-8=0$, $[x,y,z]=[7,\ 8,-1]+t[1,0,0]+s[0,0,1]$

d) $-30x+39y+123z-99=0$, $[x,y,z]=[3,8,-1]+t[-11,1,-3]+s[3,-4,2]$

e) $-7x+9y+3z+61=0$, $[x,y,z]=[4,-3,-2]+t[3,-2,1]+s[3,-1,4]$

f) $5x-4y+7z-70=0$, $[x,y,z]=[2,-1,8]+t[4,5,0]+s[0,7,4]$ **g)** $-3x+2y+5z-11=0$, $[x,y,z]=[5,-7,8]+t[2,3,0]+s[0,-5,2]$

h) $-8x-4y-4z+28=0$, $[x,y,z]=[4,-1,0]+t[-6,5,7]+s[-2,1,3]$ **4.** $53.3°$

5. $[x,y,z]=[0,0,2]+t[1,3,0]+s[0,2,1]$, parametric: $x=t, y=3t+2s, z=2+s$

6.a) perpendicular: [2,4,4]; parallel: [-2,0,1] **b)** perpendicular: [5,0,2], parallel: [-2,0,5] **c)** perpendicular: [0,5,0]; parallel: [2,0,1]

7.a) $\vec{n}=[-1,4,2]$ **b)** $S(0,0,-3)$, $T(6,0,0)$ **c)** $\overrightarrow{ST}=[6,0,3]$ **d)** $\overrightarrow{ST}\cdot\vec{n}=0$

8.a) $z=0$ **b)** $y=0$ **c)** $x=0$