W5 – Intersections of Lines in 2-Space and 3-Space Unit 6

MCV4U

Jensen

1) Solve each linear system in two-space.

a) $3x - 5y = -9, \quad 4x + 5y = 23$

b) $[x, y] = [5,4] + s[-3,1], \quad [x, y] = [2,2] + t[2, -1]$

2) Determine if the parallel lines in each pair are distinct or coincident.

a) $[x, y, z] = [5, -2, -8] + s[-3,2,5]$
$[x, y, z] = [-4,0,2] + t[-3,2,5]$

b) $[x, y, z] = [-3,0, -6] + s[-3,0, -6]$
$[x, y, z] = [9,1,18] + t[9,0,18]$

3) Triangle ABC is formed from the intersections of the three lines represented by these equations. Find the length of each side of ΔABC.

$l_1: [x, y] = [1, -3] + t[0,1]$ $l_2: [x, y] = [2,4] + s[-1,6]$ $l_3: [x, y] = [2,3] + r[1,7]$

4) Parallelogram ABCD has vertices A(–1, –4), B(1, –3), C(6, –6), and D(4, –7). Find the vector equations of its diagonals and the point of intersection of the diagonals.

5) Determine if the lines in each pair intersect. If so, find the co-ordinates of the point of intersection.

a) $[x, y, z] = [9, -1, 1] + s[-3, 4, 1]$
$[x, y, z] = [-3, 11, 5] + t[-3, 4, 1]$

b) $[x, y, z] = [1, 4, 5] + s[3, 0, -2]$
$[x, y, z] = [9, 4, -3] + t[3, 0, -2]$

c) $[x, y, z] = [1, 0, -3] + t[3, 5, 4]$
$[x, y, z] = [0, -9, -1] + s[-1, 2, -3]$

d) $[x, y, z] = [4,7,-1] + s[-2,1,2]$
$[x, y, z] = [1,3,-1] + t[4,-1,2]$

e) $[x, y, z] = [-2,0,-3] + t[5,1,3]$
$[x, y, z] = [5,8,-6] + s[-1,2,-3]$

6) Determine the distance between the skew lines in each pair.

a) $\ell_1: [x, y, z] = [4,-3,2] + s[2,7,-1]$
$\ell_2: [x, y, z] = [-2,5,4] + t[-4,0,3]$

b) $\ell_1: [x, y, z] = [-1,6,1] + s[-2,4,3]$
$\ell_2: [x, y, z] = [5,1,9] + t[3, -2,4]$

ANSWER KEY:
1. a) (2, 3) **b)** (–16, 11)
2. a) parallel and distinct **b)** parallel and distinct
3. 6.55, 14, 7.61
4. $\overrightarrow{AC}$= [–1, –4] + *s*[7, –2], $\overrightarrow{BD}$= [1, –3] + *t*[3, –4]; $\left(\frac{5}{2}, -5\right)$
5. a) No solutions; parallel and distinct **b)** No **c)** (–2, –5, –7) **d)** No **e)** (8, 2, 3)
6. a) 2.45 **b)** 0.59