W5 – Combinations of Functions

MHF4U

Jensen

1) Let $f(x) = 3x - 5$ and $g(x) = 2x + 3$.

a) Write the equation for $h(x) = f(x) + g(x)$ and determine the value of $h(2)$.

$h(x) = (3x - 5) + (2x + 3)$

$h(x) = 3x - 5 + 2x + 3$

$h(x) = 5x - 2$

$h(2) = 5(2) - 2$

$h(2) = 8$

b) Write the equation for $k(x) = f(x) - g(x)$ and determine the value of $k(2)$.

$k(x) = (3x - 5) - (2x + 3)$

$k(x) = 3x - 5 - 2x - 3$

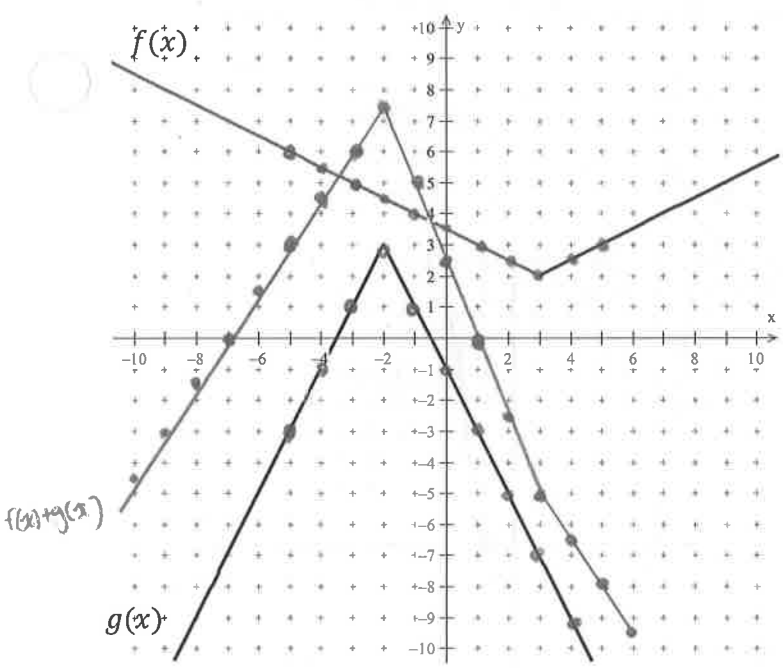
$k(x) = x - 8$

$k(2) = 2 - 8$

$= -6$

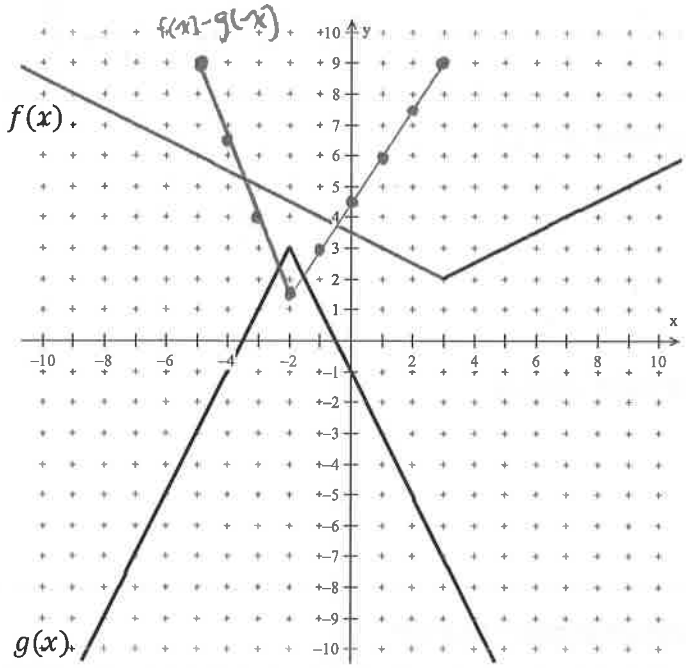
2) Use the functions $f(x)$ and $g(x)$ as shown. Apply the superposition principle to graph

a) $y = f(x) + g(x)$



x	$f(x)$	$g(x)$	$f(x)+g(x)$	$f(x)-g(x)$
-5	6	-3	3	9
-4	5.5	-1	4.5	6.5
-3	5	1	6	4
-2	4.5	3	7.5	1.5
-1	4	1	5	3
0	3.5	-1	2.5	4.5
1	3	-3	0	6
2	2.5	-5	-2.5	7.5
3	2	-7	-5	9
4	2.5	-9	-6.5	11.5
5	3	-11	-8	14

b) $y = f(x) - g(x)$.



3) Let $f(x) = x - 2$ and $g(x) = x^2 + 3x - 3$. Determine an algebraic and graphical model for $h(x) = f(x) + g(x)$.

$h(x) = (x-2) + (x^2+3x-3)$

$h(x) = x - 2 + x^2 + 3x - 3$

$h(x) = x^2 + 4x - 5$

$h(x) = (x+5)(x-1)$

x-int: $x = -5$ and $x = 1$

x-vertex at $\frac{-5+1}{2} = -2$

$h(-2) = (-2)^2 + 4(-2) - 5$

$= -9$

$(-2, -9)$

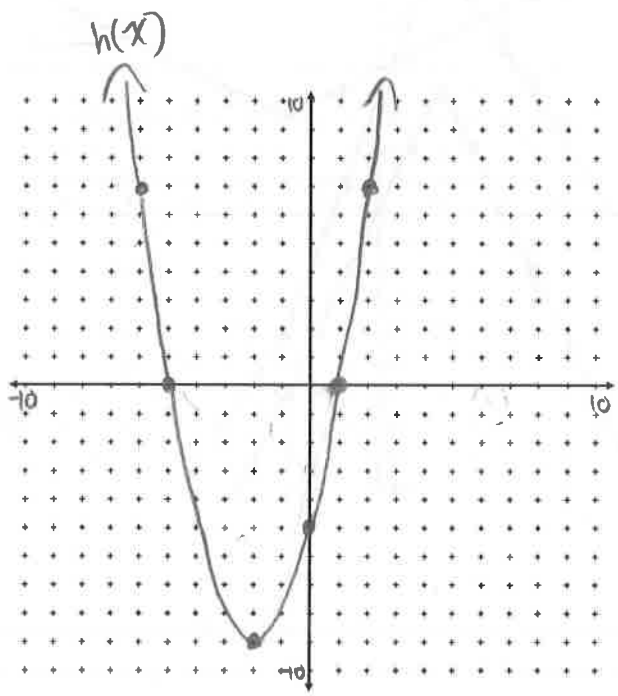
y-int:

$h(0) = 0^2 + 4(0) - 5$

$h(0) = -5$

$(0, -5)$

x	y
-6	7
-5	0
-2	-9
1	0
2	7



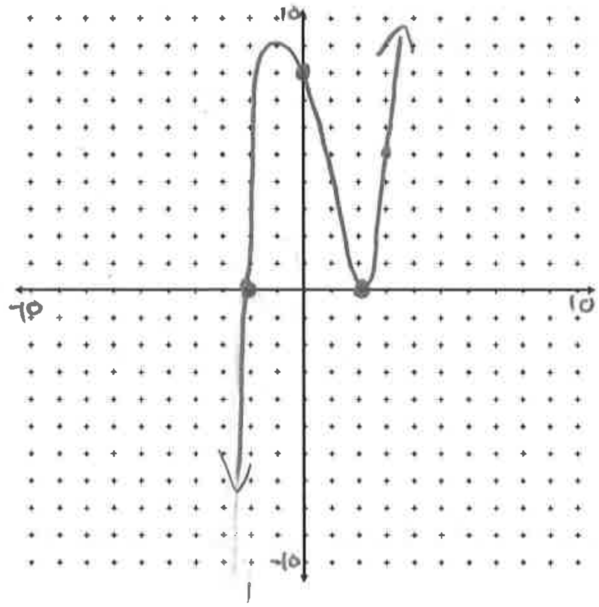
4) Let $f(x) = x - 2$ and $g(x) = x^2 - 4$. Develop an algebraic and graphical model for each of the following:

a) $y = f(x)g(x)$

$$= (x-2)(x^2-4)$$
$$= (x-2)(x-2)(x+2)$$
$$= (x-2)^2(x+2)$$

x-int: $(2,0)$ order 2; $(-2,0)$ order 1

y-int: $y = (0-2)^2(0+2)$; $y = 8$; $(0,8)$



b) $y = \frac{f(x)}{g(x)}$

$$= \frac{x-2}{x^2-4}$$
$$= \frac{\cancel{x-2}}{(\cancel{x-2})(x+2)}$$
$$= \frac{1}{x+2}; \; x \neq -2, 2$$

Hole at $x=2$

VA: $x=-2$

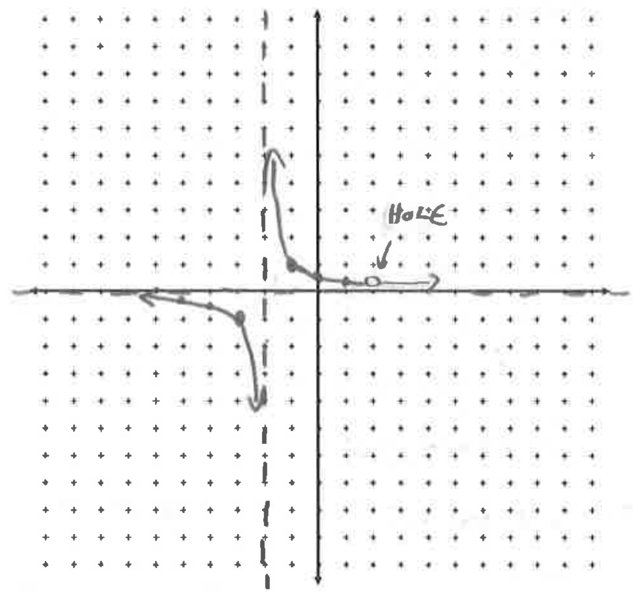
HA: $y=0$

$x+2$

x	y
-5	-3
-4	-2
-3	-1
-2	0
-1	1
0	2
1	3

$\frac{1}{x+2}$

x	y
-5	-0.33
-4	-0.5
-3	-1
-2	und.
-1	1
0	0.5
1	0.33

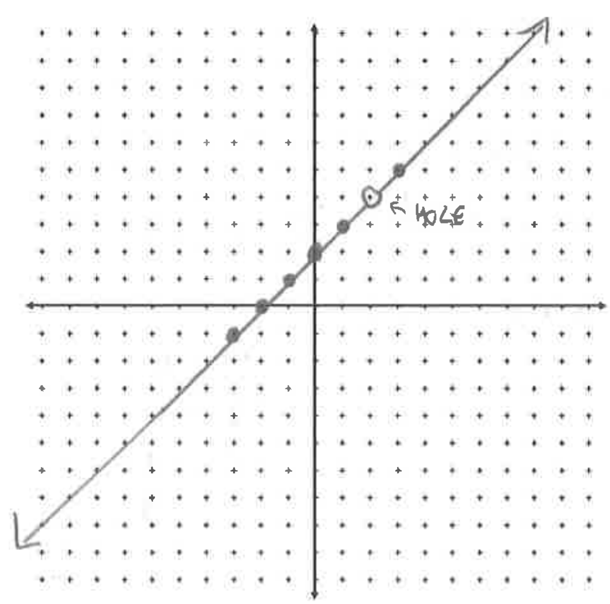


c) $y = \frac{g(x)}{f(x)}$

$$= \frac{x^2-4}{x-2}$$
$$= \frac{(\cancel{x-2})(x+2)}{\cancel{x-2}}$$
$$= x+2; \; x \neq 2$$

Hole at $x=2$

x	y
-3	-1
-2	0
-1	1
0	2
1	3
2	undefined
3	5



5) Let $f(x) = x^2 + 2x - 4$ and $g(x) = \frac{1}{x+1}$.

a) Evaluate $g(f(0))$

$f(0) = (0)^2 + 2(0) - 4$

$f(0) = -4$

$g(f(0)) = g(-4)$

$= \frac{1}{-4+1}$

$\boxed{= -\frac{1}{3}}$

b) Evaluate $f(g(-2))$

$g(-2) = \frac{1}{-2+1}$

$g(-2) = -1$

$f(g(-2)) = f(-1)$

$= (-1)^2 + 2(-1) - 4$

$\boxed{= -5}$

6) Let $f(x) = x^2 + 3x$ and $g(x) = 2x - 5$. Determine an equation for each composite function and graph it.

a) $y = f(g(x))$

$f(g(x)) = (2x-5)^2 + 3(2x-5)$

$= 4x^2 - 20x + 25 + 6x - 15$

$= 4x^2 - 14x + 10$

$= 2(2x^2 - 7x + 5)$

$= 2[2x^2 - 5x - 2x + 5]$

$= 2[x(2x-5) - 1(2x-5)]$

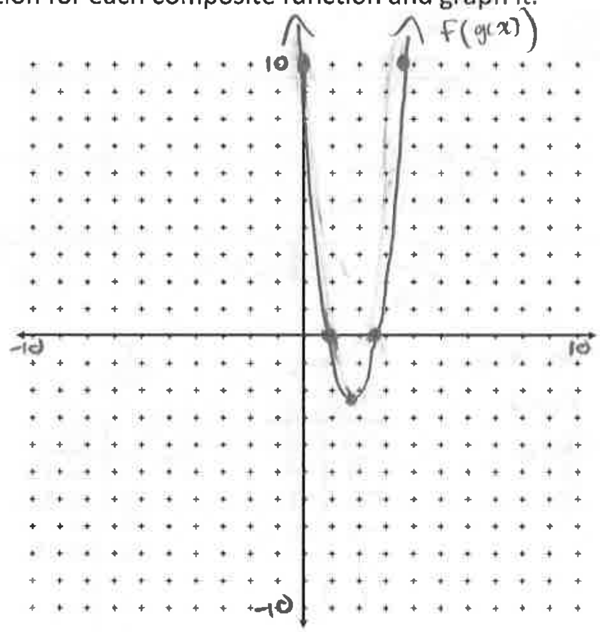
$= 2(2x-5)(x-1)$

x-int: $x = 1$ and $x = 2.5$

y-int: $(0, 10)$

x-vertex at $\frac{1+2.5}{2} = 1.75$

y-vertex $= -2.025$



b) $y = g(f(x))$

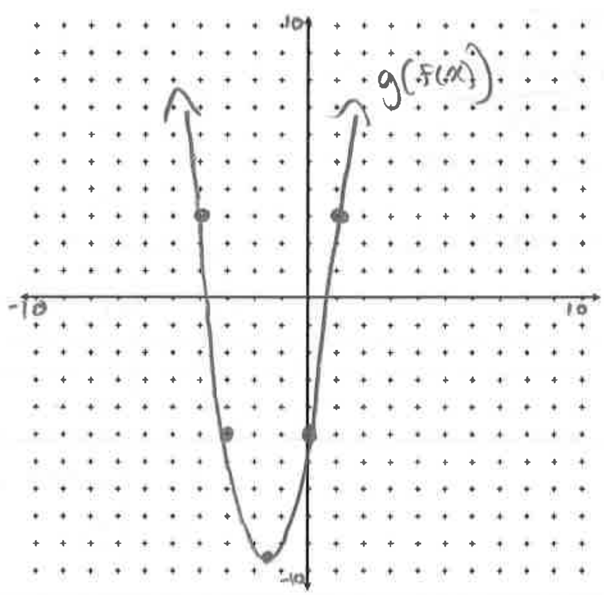
$g(f(x)) = 2(x^2+3x) - 5$

$= 2x^2 + 6x - 5$

x-vertex at $\frac{-b}{2a} = \frac{-6}{2(2)} = -1.5$

y-vertex $= -9.5$

x	y
-4	3
-3	-5
-1.5	-9.5
0	-5
1	3



c) $y = g(g(x))$

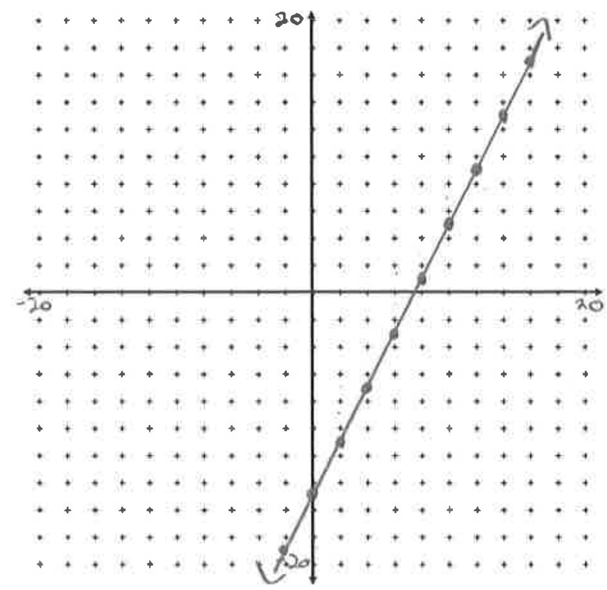
$= 2(2x-5)-5$

$= 4x-10-5$

$= 4x-15$

$m=4$

$b=-15$



$y = g^{-1}(g(x))$

$g^{-1}(x)$:

$y=2x-5$

$x=2y-5$

$\frac{x+5}{2}=y$

$g^{-1}(x) = \frac{x+5}{2}$

$g^{-1}(g(x)) = \frac{(2x-5)+5}{2}$

$= \frac{2x}{2}$

$= x$

$m=1$

$b=0$