

**W6 – 6.5 – Applications of Logarithms AND Exponentials in Physical Sciences**

MHF4U

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***Exponential Formulas***

$$A(t) = A_0(1+i)^t$$

general, where  $i$  is percent growth(+) or decay(-)

$$A(t) = A_0\left(\frac{1}{2}\right)^{\frac{t}{H}}$$

half-life,  $H$  is the half-life period

$$A(t) = A_0(2)^{\frac{t}{D}}$$

doubling,  $D$  is the doubling period

***Logarithmic Formulas***

$$pH = -\log[H^+]$$

Where pH is acidity and  $[H^+]$  is concentration of hydronium ions mol/L

$$\beta_2 - \beta_1 = 10 \log\left(\frac{I_2}{I_1}\right)$$

Where  $\beta$  is loudness in dB and  $I$  is intensity of sound in  $W/m^2$

$$M = \log\left(\frac{I}{I_0}\right)$$

Where  $M$  is magnitude measure by richters,  $I$  is intensity

**1)** The half-life of a radioactive form of tritium is about 2 years. How much of a 5-kg sample of this material would remain after ...

**a)** 8 years

**b)** 12 months

**2)** The population of Littleton is currently (2014) 23000, and is increasing exponentially with a growth rate of 2% per year. Estimate when Littleton will have a population of 30000.

**3)** The population of purple martins in Algonquin park was estimated to be 35000 in 1992. Ten years later, in 2002, the population had risen to 44400.

**a)** What is the annual growth rate for the purple martin population?

**b)** Estimate the population for 2010 to the nearest hundred.

**4)** After an accident at a nuclear plant, the radiation level in the plant was 950 R (roentgens). Five hours later the level was 800 R. How long will it take before safe levels of radiation are reached, which is less than 0.01 R?

**5)** The value of a new minivan drops 40% after the first year, and then decreases exponentially at a rate of 12% per year after that. When will a minivan that cost \$35000 new be worth less than \$10000?

**6)** A crab fossil contains 38.6% of its original Carbon14 isotope, which has a half life of 5370 years. Approximately how old is the crab fossil?

**7)** A Trimark mutual fund has track record of 4.2% growth per year. What is the doubling period for this investment?

**8)** A treatment to help a patient stop smoking involves chewing nicotine gum. Each gum introduces 1.5 mg of nicotine into the patient's system. Nicotine has a half life of 3 hours. The patient will feel the urge to smoke when the level of nicotine drops below 0.45 mg in her system. If she chewed a gum at 8:00 a.m., and another at 10:00 a.m., at what time will she next feel the urge to smoke?

**9)** Determine the pH of a solution with hydronium ion concentration:

**a)** 0.01

**b)**  $1.5 \times 10^{-10}$

**10)** Determine the hydronium concentration, in moles per litre, of a solution with pH:

**a)** 8.5

**b)** 3

**11)** How many times as intense is the sound of a shout as the sound of a whisper?

**12)** A loud car stereo has a decibel level of 110 dB. How many times as intense as the sound of a loud car stereo is the sound of a rock concert speaker?

**13)** The sound intensity of a pin drop is about  $1/30\,000$  of the sound intensity of a normal conversation. What is the decibel level of a pin drop?

**14)** On September 26, 2001, an earthquake in North Bay, Ontario, occurred that was 10 000 times as intense as  $I_0$ . What was the measure of this earthquake on the Richter scale?

**15)** On February 10, 2000, an earthquake happen in Welland, Ontario, that measured 2.3 on the Richter scale.  
**a)** How many times as intense was this as a standard low-level earthquake?

**b)** On July 22, 2001, an earthquake in St. Catharines measured 1.1 on the Richter scale. How many times as intense as the St.Catharines earthquake was the Welland earthquake?

**16)** The stellar magnitude scale compares the brightness of stars using the equation  $m_2 - m_1 = \log\left(\frac{b_1}{b_2}\right)$ , where  $m_2$  and  $m_1$  are the apparent magnitude of the two stars being compared (how bright they appear in the sky) and  $b_1$  and  $b_2$  are their brightness (how much light they actually emit). This relationship does not factor in how far from Earth the stars are.

**a)** Sirius is the brightest-appearing star in our sky, with an apparent magnitude of -1.5. How much brighter does Sirius appear than Betelgeuse, whose apparent magnitude is 0.12?

**b)** The Sun appears about  $1.3 \times 10^{10}$  times as brightly in our sky as does Sirius. What is the apparent magnitude of the Sun?

**ANSWER KEY – Exponentials**

**1. a)** 8 years      **b)** 12 months  
0.3125 kg      3.536 kg

**2.** In approx. 13.418 years

**3. a)** Growth rate of 2.4% per year  
**3. b)** Approx. 53 700 birds

**4.** Approx. 333 hours

**5.** Approx. 7 years

**6.** Approx. 7375 years

**7.** Approx. 16.85 years

**8.**  $t \sim 7.3$  hours, so about 5:20 p.m

**ANSWER KEY: Logarithms**

**9. a)** 2      **b)** 9.8

**10. a)**  $3.2 \times 10^{-9}$  mol/L  
**b)** 0.001 mol/L

**11.** A shout is 100,000 times more intense than a whisper

**12.** A rock concert speaker is 10,000 times more intense than a loud car speaker

**13.** A pin drop is 15dB

**14.** level 4 on the Richter scale

**15. a)** about 200 times more intense  
**b)** 15.85 times more intense

**16. a)** 41.69 times brighter  
**b)** apparent magnitude is -11.61