W6 – Intersection of Lines and Planes Unit 6

MCV4U

Jensen

1. In each case, determine if the line and the plane are parallel. If so, decide on distinct or coincident.

a) $l_1: \begin{cases} x = 4 + 2t \\ y = -t \\ z = -1 - 4t \end{cases}$ $\pi_1: 3x + 2y + z - 7 = 0$

b) $l_2: \begin{cases} x = t \\ y = 2t \\ z = 3t \end{cases}$ $\pi_2: x - y + 2z = 5$

2. In each case, determine if the plane and line intersect. If so, state the solution.

a) $[x, y, z] = [1,2,5] + t[1, -1,2]$ $2x + 6y - z = 5$

b) $[x, y, z] = [6,11,1] + t[1,5,2]$ $x + 3y + 2z - 1 = 0$

c) $[x, y, z] = [9,8,3] + t[2,1,5]$ $z = 0$

d) $[x, y, z] = [4,2,6] + t[1, -2,3]$ $2x + 5y - z - 34 = 0$

e) $[x, y, z] = [3,2,-1] + t[-2,1,3]$ $\quad\quad\quad\quad$ $x + 2y - 3z = 10$

f) $[x, y, z] = [4,2,6] + t[1,-2,3]$ $\quad\quad\quad\quad$ $-4x - 5y + 6z = 34$

3. Determine if each of the following lines intersects the plane:
$[x, y, z] = [4, -15, -8] + s[1, -3, 1] + t[2, 3, 1]$. If so, how many intersections are there?

a) $[x, y, z] = [5, -9, 3] + k[1, -12, 2]$ $\quad\quad\quad\quad$ **b)** $[x, y, z] = [-2, 9, -21] + k[2, -5, 4]$

c) $[x, y, z] = [3, -2, 1] + k[1, 4, -2]$

d) $[x, y, z] = [4, -24, -7] + k[-2, -3, -1]$

e) $[x, y, z] = [2, -3, 0] + k[-1, 3, -1]$

f) $[x, y, z] = [9, 4, 1] + k[-2, 2, 4]$

4. Find the distance between each point and the given plane.

a) $P(1,1,-1)$, $x + y - z - 3 = 0$

b) $P(7,-3,2)$, $2x - 3z - 1 = 0$

5. Find the distance between the planes $\pi_1: 2x + 2y - z - 3 = 0$ and $\pi_2: 4x + 4y - 2z + 9 = 0$

ANSWER KEY:

1. a) Yes (distinct) **b)** No **2. a)** $\left(\frac{5}{3}, \frac{4}{3}, \frac{19}{3}\right)$ **b)** (4, 1, –3) **c)** $\left(\frac{39}{5}, \frac{37}{5}, 0\right)$ **d)** (2, 6, 0) **e)** (3, 2, -1) **f)** (5, 0, 9)

3. a) No **b)** Yes – one **c)** Yes – one **d)** Yes – infinite **e)** No **f)** Yes – one

4.a) 0 **b)** 1.94

5. 2.5