W6 – Intersection of Lines and Planes Unit 6
MCV4U
Jensen

1. In each case, determine if the line and the plane are parallel. If so, decide on distinct or coincident.

a) $l_1: \begin{cases} x = 4 + 2t \\ y = -t \\ z = -1 - 4t \end{cases}$ $\pi_1: 3x + 2y + z - 7 = 0$

$\vec{m} = [2,-1,-4]$

$\vec{n} = [3,2,1]$

$\vec{m} \cdot \vec{n} = 2(3) + (-1)(2) + (-4)(1)$

$= 0$

$\therefore \vec{m}$ and $\vec{n}$ are perpendicular

$\therefore l_1$ is parallel to the plane.

Check if $(4,0,-1)$ is on the plane:

LS

$= 3(4) + 2(0) + (-1) - 7$

$= 4$

RS

$= 0$

$LS \neq RS$

$\therefore$ parallel and distinct.

b) $l_2: \begin{cases} x = t \\ y = 2t \\ z = 3t \end{cases}$ $\pi_2: x - y + 2z = 5$

$\vec{m} = [1,2,3]$

$\vec{n} = [1,-1,2]$

$\vec{m} \cdot \vec{n} = 1(1) + 2(-1) + 3(2)$

$= 5$

$\therefore \vec{m}$ and $\vec{n}$ are not perpendicular

$\therefore l_2$ is NOT parallel to π_2.

2. In each case, determine if the plane and line intersect. If so, state the solution.

a) $[x, y, z] = [1,2,5] + t[1,-1,2]$ $2x + 6y - z = 5$

$l \begin{cases} x = 1 + t \\ y = 2 - t \\ z = 5 + 2t \end{cases}$

$2(1+t) + 6(2-t) - (5+2t) = 5$

$2 + 2t + 12 - 6t - 5 - 2t = 5$

$-6t + 9 = 5$

$-6t = -4$

$t = \frac{2}{3}$

POI

$x = 1 + \frac{2}{3} = \frac{5}{3}$

$y = 2 - \frac{2}{3} = \frac{4}{3}$

$z = 5 + 2\left(\frac{2}{3}\right) = \frac{19}{3}$

$\left(\frac{5}{3}, \frac{4}{3}, \frac{19}{3}\right)$

b) $[x, y, z] = [6,11,1] + t[1,5,2]$ $\quad x + 3y + 2z - 1 = 0$

$l: \begin{cases} x = 6+t \\ y = 11+5t \\ z = 1+2t \end{cases}$

$(6+t) + 3(11+5t) + 2(1+2t) - 1 = 0$

$6+t+33+15t+2+4t-1=0$

$20t + 40 = 0$

$t = -2$

POI

$x = 6+(-2) = 4$

$y = 11+5(-2) = 1$

$z = 1+2(-2) = -3$

$(4,1,-3)$

c) $[x, y, z] = [9,8,3] + t[2,1,5]$ $\quad z = 0$

$l \begin{cases} x = 9+2t \\ y = 8+t \\ z = 3+5t \end{cases}$

$3+5t = 0$

$t = \frac{-3}{5}$

POI

$x = 9+2\left(\frac{-3}{5}\right) = \frac{39}{5}$

$y = 8+\left(\frac{-3}{5}\right) = \frac{37}{5}$

$z = 0$

$\left(\frac{39}{5}, \frac{37}{5}, 0\right)$

d) $[x, y, z] = [4,2,6] + t[1,-2,3]$ $\quad 2x + 5y - z - 34 = 0$

$l \begin{cases} x = 4+t \\ y = 2-2t \\ z = 6+3t \end{cases}$

$2(4+t) + 5(2-2t) - (6+3t) - 34 = 0$

$8+2t+10-10t-6-3t-34=0$

$-11t = 22$

$t = -2$

POI

$x = 4+(-2) = 2$

$y = 2-2(-2) = 6$

$z = 6+3(-2) = 0$

$(2,6,0)$

e) $[x, y, z] = [3,2,-1] + t[-2,1,3]$ $\quad x + 2y - 3z = 10$

$l: \begin{cases} x = 3 - 2t \\ y = 2 + t \\ z = -1 + 3t \end{cases}$

$(3-2t) + 2(2+t) - 3(-1+3t) = 10$

$3 - 2t + 4 + 2t + 3 - 9t = 10$

$-9t = 0$

$t = 0$

POI

$x = 3 - 2(0) = 3$

$y = 2 + 0 = 2$

$z = -1 + 3(0) = -1$

$(3, 2, -1)$

f) $[x, y, z] = [4,2,6] + t[1,-2,3]$ $\quad -4x - 5y + 6z = 34$

$l: \begin{cases} x = 4 + t \\ y = 2 - 2t \\ z = 6 + 3t \end{cases}$

$-4(4+t) - 5(2-2t) + 6(6+3t) = 34$

$-16 - 4t - 10 + 10t + 36 + 18t = 34$

$24t = 24$

$t = 1$

POI

$x = 4 + 1 = 5$

$y = 2 - 2(1) = 0$

$z = 6 + 3(1) = 9$

$(5, 0, 9)$

3. Determine if each of the following lines intersects the plane:
π: $[x, y, z] = [4, -15, -8] + s[1, -3, 1] + t[2, 3, 1]$. If so, how many intersections are there?

($\vec{m}_1$, $\vec{m}_2$)

a) $[x, y, z] = [5, -9, 3] + k[1, -12, 2]$ ($\vec{m}_3$)

$\vec{n} = \vec{m}_1 \times \vec{m}_2 = [-6, 1, 9]$

$\begin{matrix} -3 & 3 \\ 1 & 1 \\ 1 & 2 \\ -3 & 3 \end{matrix}$

$\vec{m}_3 \cdot \vec{n} = 1(-6) + (-12)(1) + 2(9)$

$= 0$

∴ The line is parallel to the plane.

π: $-6x + y + 9z + D = 0$

$-6(4) + (-15) + 9(-8) + D = 0$

$D = 111$

$-6x + y + 9z + 111 = 0$

$-6(5) + (-9) + 9(3) + 111 = 0$

$99 = 0$

∴ P(5, -9, 3) not on plane.

Parallel and distinct

b) $[x, y, z] = [-2, 9, -21] + k[2, -5, 4]$ ($\vec{m}_4$)

$\vec{n} \cdot \vec{m}_4 = -6(2) + 1(-5) + 9(4)$

$= 19$

∴ the line is not parallel to the plane.

There is 1 point of intersection.

c) $[x,y,z] = [3,-2,1] + k[1,4,-2]$ $\quad \vec{m}_5$

$\vec{n} \cdot \vec{m}_5 = -6(1) + 1(4) + 9(-2)$

$= -20$

∴ line and plane are not parallel

1 point of intersection.

d) $[x,y,z] = [4,-24,-7] + k[-2,-3,-1]$ $\quad \vec{m}_6$

$\vec{n} \cdot \vec{m}_6 = -6(-2) + 1(-3) + 9(-1)$

$= 0$

∴ line and plane are parallel

$-6(4) + (-24) + 9(-7) + 111 = 0$

$0 = 0$

∴ point is on plane. Infinite solutions.

e) $[x,y,z] = [2,-3,0] + k[-1,3,-1]$ $\quad \vec{m}_7$

$\vec{n} \cdot \vec{m}_7 = -6(-1) + 1(3) + 9(-1)$

$= 0$

∴ line and plane are parallel

$-6(2) + (-3) + 9(0) + 111 = 0$

$96 = 0$

∴ point NOT on plane. Parallel and distinct

f) $[x,y,z] = [9,4,1] + k[-2,2,4]$ $\quad \vec{m}_8$

$\vec{n} \cdot \vec{m}_8 = -6(-2) + 1(2) + 9(4)$

$= 50$

∴ line and plane are NOT parallel.

∴ 1 point of intersection.

4. Find the distance between each point and the given plane.

a) $P(1,1,-1)$, $\quad x + y - z - 3 = 0$

Point Q on plane:

$0 + 0 - z - 3 = 0$

$z = -3$

$Q(0,0,-3)$

$\vec{PQ} = [-1,-1,-2]$

$\vec{n} = [1,1,-1]$

$$d = \frac{|\vec{PQ} \cdot \vec{n}|}{|\vec{n}|}$$

$$= \frac{-1(1) + (-1)(1) + (-2)(-1)}{\sqrt{(1)^2 + (1)^2 + (-1)^2}}$$

$$= \frac{0}{\sqrt{3}}$$

$= 0$ ∴ The point is on the plane.

b) $P(7,-3,2)$, $\quad 2x - 3z - 1 = 0$

Point Q on plane:

$2(2) - 3z - 1 = 0$

$z = 1$

$Q(2,5,1)$

$\vec{PQ} = [-5,8,-1]$

$\vec{n} = [2,0,-3]$

$$d = \frac{|-5(2) + 8(0) + (-1)(-3)|}{\sqrt{(2)^2 + (0)^2 + (-3)^2}}$$

$$d = \frac{7}{\sqrt{13}}$$

$d \simeq 1.94$

5. Find the distance between the planes $\pi_1: 2x + 2y - z - 3 = 0$ and $\pi_2: 4x + 4y - 2z + 9 = 0$

Point on π_1: $2(0) + 2(0) - z - 3 = 0$

$z = -3$

$A(0, 0, -3)$

Point on π_2: $4(0) + 4(0) - 2z + 9 = 0$

$z = 4.5$

$B(0, 0, 4.5)$

$\vec{AB} = [0, 0, 7.5]$

$\vec{n} = [2, 2, -1]$

$$d = \frac{|0(2) + 0(2) + 7.5(-1)|}{\sqrt{(2)^2 + (2)^2 + (-1)^2}}$$

$$= \frac{|-7.5|}{3}$$

$$= 2.5$$

ANSWER KEY:

1. a) Yes (distinct) **b)** No **2. a)** $\left(\frac{5}{3}, \frac{4}{3}, \frac{19}{3}\right)$ **b)** (4, 1, –3) **c)** $\left(\frac{39}{5}, \frac{37}{5}, 0\right)$ **d)** (2, 6, 0) **e)** (4, 8, 3) **f)** (5, 0, 9) (3, 2, -1)

3. a) No **b)** Yes – one **c)** Yes – one **d)** Yes – infinite **e)** No **f)** Yes – one

4.a) 0 **b)** 1.94

5. 2.5