W7 – Intersection of Planes Unit 6

MCV4U

Jensen

Determine the solution(s) to each system of planes. If there are no consistent solutions, give evidence and or an explanation of why.

a) $\pi_1: 3x - 2y + z = 4$
$\pi_2: 6x - 4y + 3z = 7$

b) $\pi_1: 2x - 8y - 6z - 2 = 0$
$\pi_2: -x + 4y + 3z - 5 = 0$

c) $\pi_1: x - 2y + 2.5z - 1 = 0$
$\pi_2: 3x - 6y + 7.5z - 3 = 0$

d) $\pi_1: x + y + 2z = 5$
$\pi_2: 4x - 3y + z = -8$
$\pi_3: -5x - 2y + 3z = 7$

e) $\pi_1: x + y + z = -3$
$\pi_2: 2x + 2y - 3z = 4$
$\pi_3: 3x + 3y - 2z = 1$

f) $\pi_1: -x + y + 3z = 2$
$\pi_2: 2x - 2y - 6z = -4$
$\pi_3: -3x + 3y + 9z = 6$

g) $\pi_1: x - 4y - 13z = 4$
$\pi_2: x - 2y - 3z = 2$
$\pi_3: -3x + 5y + 4z = 2$

h) $\pi_1: 3x - 2y + 5z = 1$
$\pi_2: 5x + y - 3z = -4$
$\pi_3: x - 5y + 13z = 6$

i) $\pi_1: -x + y + 3z = 2$
$\pi_2: -x + y + 3z = 4$
$\pi_3: x - 3y + 5z = 6$

j) $\pi_1: x + 4y + 3z = 5$
$\pi_2: x + 3y + 2z + 4 = 0$
$\pi_3: x + y - z = -1$

Answers:

a) $[x, y, z] = \left[\frac{5}{3}, 0, -1\right] + t\left[\frac{2}{3}, 1, 0\right]$
b) No solutions; parallel and distinct
c) Infinite solutions, parallel and coincident
d) (-1,2,2)
e) $[x, y, z] = [-1, 0, -2] + s[-1, 1, 0]$
f) infinite solutions; three coincident planes
g) inconsistent system; the planes intersect in pairs
h) $[x, y, z] = \left[-\frac{7}{13}, -\frac{17}{13}, 0\right] + t\left[\frac{1}{13}, \frac{34}{13}, 1\right]$
i) inconsistent system; two parallel and distinct planes intersected by a third plane
j) (-52,30,-21)