W7 – Intersection of Planes Unit 6

MCV4U

Jensen

SOLUTIONS

Determine the solution(s) to each system of planes. If there are no consistent solutions, give evidence and or an explanation of why.

a) $\pi_1: 3x - 2y + z = 4$
$\pi_2: 6x - 4y + 3z = 7$

b) $\pi_1: 2x - 8y - 6z - 2 = 0$
$\pi_2: -x + 4y + 3z - 5 = 0$

c) $\pi_1: x - 2y + 2.5z - 1 = 0$
$\pi_2: 3x - 6y + 7.5z - 3 = 0$

d) $\pi_1: x + y + 2z = 5$
$\pi_2: 4x - 3y + z = -8$
$\pi_3: -5x - 2y + 3z = 7$

e) $\pi_1: x + y + z = -3$
$\pi_2: 2x + 2y - 3z = 4$
$\pi_3: 3x + 3y - 2z = 1$

f) $\pi_1: -x + y + 3z = 2$
$\pi_2: 2x - 2y - 6z = -4$
$\pi_3: -3x + 3y + 9z = 6$

g) $\pi_1: x - 4y - 13z = 4$
$\pi_2: x - 2y - 3z = 2$
$\pi_3: -3x + 5y + 4z = 2$

h) $\pi_1: 3x - 2y + 5z = 1$
$\pi_2: 5x + y - 3z = -4$
$\pi_3: x - 5y + 13z = 6$

i) $\pi_1: -x + y + 3z = 2$
$\pi_2: -x + y + 3z = 4$
$\pi_3: x - 3y + 5z = 6$

j) $\pi_1: x + 4y + 3z = 5$
$\pi_2: x + 3y + 2z + 4 = 0$
$\pi_3: x + y - z = -1$

a) normals are not parallel; ∴ intersect in a line.

$2 \times ①\quad 6x - 4y + 2z = 8$
$\quad\ \ 6x - 4y + 3z = 7 \quad -$

$-z = 1$
$z = -1$

Let $x = t$

$3t - 2y + (-1) = 4$
$3t - 1 - 4 = 2y$
$3t - 5 = 2y$
$\frac{3}{2}t - \frac{5}{2} = y$

$l: \begin{cases} x = t \\ y = \frac{3}{2}t - \frac{5}{2} \\ z = -1 \end{cases}$

$[x, y, z] = [0, -2.5, -1] + t[1, 1.5, 0]$

b) normal ARE parallel; check if coincident or distinct.

$①\quad 2x - 8y - 6z - 2 = 0$
$2 \times ②\quad -2x + 8y + 6z - 10 = 0 \quad +$

$-12 = 0$

∴ NO solutions

Parallel and distinct.

c) normals are parallel; check if distinct or coincident

$3\times$ ① $3x-6y+7.5z-3=0$

② $3x-6y+7.5z-3=0$ $-$

$0=0$

$\therefore$ parallel and coincident

$\therefore$ infinite solutions.

d) no normals are parallel

① $x+y+2z=5$

$2\times$ ② $8x-6y+2z=-16$ $-$

$-7x+7y=21$

$3\times$ ① $3x+3y+6z=15$

$2\times$ ③ $-10x-4y+6z=14$ $-$

$13x+7y=1$

$-7x+7y=21$

$13x+7y=1$ $-$

$-20x=20$

$x=-1$

$13x+7y=1$

$13(-1)+7y=1$

$7y=14$

$y=2$

$x+y+2z=5$

$-1+2+2z=5$

$2z=4$

$z=2$

The intersect at the point $(-1,2,2)$

e)

$2\times$ ① $2x+2y+2z=-6$

② $2x+2y-3z=4$ −

$5z=-10$

$z=-2$

$3\times$① $3x+3y+3z=-9$

③ $3x+3y-2z=1$ −

$5z=-10$

$z=-2$

let $x=t$

① $x+y+z=-3$

$t+y+(-2)=-3$

$y=-1-t$

$$l: \begin{cases} x=t \\ y=-1-t \\ z=-2 \end{cases}$$

$$[x,y,z]=[0,-1,-2]+t[1,-1,0]$$

f) $-2\pi_1=\pi_2$ Three coincident planes.

$3\pi_1=\pi_3$ Infinite solutions.

g) No parallel normals

① $x-4y-13z=4$

② $x-2y-3z=2$ −

$-2y-10z=2$

$3\times$① $3x-12y-39z=12$

③ $-3x+5y+4z=2$ +

$-7y-35z=14$

$7\times(①-②)$ $-14y-70z=14$

$2\times(3\times①+③)$ $-14y-70z=28$

$0=42$

∴ No solutions. It is an inconsistent system.

h)

$5\times① \quad 15x - 10y + 25z = 5$

$3\times② \quad 15x + 3y - 9z = -12 \quad -$

$④ \quad -13y + 34z = 17$

$① \quad 3x - 2y + 5z = 1$

$3\times③ \quad 3x - 15y + 39z = 18 \quad -$

$⑤ \quad 13y - 34z = -17$

$④ \quad -13y + 34z = 17$

$⑤ \quad 13y - 34z = -17$

$0 = 0$

∴ they intersect in a line. Let $y = t$

$-13t + 34z = 17$

$34z = 17 + 13t$

$z = \frac{17}{34} + \frac{13}{34}t$

$③ \quad x - 5y + 13z = 6$

$x - 5t + 13\left(\frac{17}{34} + \frac{13}{34}t\right) = 6$

$x - 5t + \frac{221}{34} + \frac{169}{34}t = 6$

$x = -\frac{1}{2} + \frac{1}{34}t$

$$l: \begin{cases} x = \frac{-1}{2} + \frac{1}{34}t \\ y = t \\ z = \frac{17}{34} + \frac{13}{34}t \end{cases}$$

$[x, y, z] = \left[\frac{-1}{2}, 0, \frac{17}{34}\right] + t\left[\frac{1}{34}, 1, \frac{13}{34}\right]$

i) $\vec{n}_1 = \vec{n}_2$ but $\pi_1 \neq k\pi_2$; ∴ parallel and distinct.

∴ an inconsistent system.

j) on back.

Answers:

a) $[x, y, z] = \left[\frac{5}{3}, 0, -1\right] + t\left[\frac{2}{3}, 1, 0\right]$
b) No solutions; parallel and distinct
c) Infinite solutions, parallel and coincident
d) (-1,2,2)
e) $[x, y, z] = [-1, 0, -2] + s[-1, 1, 0]$
f) infinite solutions; three coincident planes
g) inconsistent system; the planes intersect in pairs
h) $[x, y, z] = \left[-\frac{7}{13}, -\frac{17}{13}, 0\right] + t\left[\frac{1}{13}, \frac{34}{13}, 1\right]$
i) Inconsistent system; two parallel and distinct planes intersected by a third plane
j) [redacted] (-52, 30, -21)

j) (1) $x+4y+3z=5$
(2) $x+3y+2z=-4$ $-$

(4) $y+z=9$

(1) $x+4y+3z=5$
(3) $x+y-z=-1$ $-$

(5) $3y+4z=6$

$3\times$(4) $3y+3z=27$
(5) $3y+4z=6$ $-$

$-z=21$
$z=-21$

∴ intersect at a point

(4) $y+(-21)=9$
$y=30$

(1) $x+4(30)+3(-21)=5$
$x=-52$

They intersect at $(-52, 30, -21)$

$10=6+3t$
$0=-2-t$
$z=-3-2(-2)$

$(3,0,0)$
$(0,0,1)$