Trigonometry of Acute Triangles

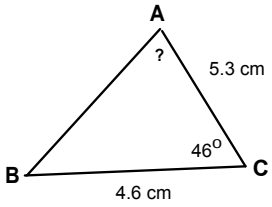
Cosine Law

$$a^2 = b^2 + c^2 - 2bc\cos(A)$$
$$b^2 = a^2 + c^2 - 2ac\cos(B)$$
$$c^2 = a^2 + b^2 - 2ab\cos(C)$$

DO IT NOW!

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Use the Sine Law to solve for the unknown angle.



Need the cosine law!

Sine Law only works to find:

1) an unknown side if two angles and a side are known

2) an unknown angle if two sides and the angle opposite one of the known sides are known

Note: The DO IT NOW question does not meet the criteria of either of these situations. There is an equation that allows us to solve for an unknown side when two sides and a contained angle are known.

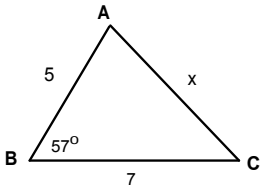
So what do we do? **Use the cosine law!!!!!**

$$a^2 = b^2 + c^2 - 2bc\cos(A)$$
$$b^2 = a^2 + c^2 - 2ac\cos(B)$$
$$c^2 = a^2 + b^2 - 2ab\cos(C)$$

See handout for full proof

When can we use the cosine law?

1) to find a missing side of an acute triangle if the other two sides and their contained angle are known



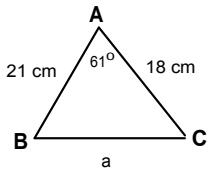
and tomorrow we will learn how to rearrange the cosine law to:

2) find an angle if you know three side lengths of an acute triangle

Today: Use the cosine law to find a missing side **of an acute triangle** if the other two sides and their contained angle are known

1 Find the missing side length

$a^2 = b^2 + c^2 - 2bc\cos(A)$
$b^2 = a^2 + c^2 - 2ac\cos(B)$
$c^2 = a^2 + b^2 - 2ab\cos(C)$

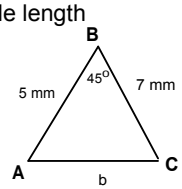


$a = ?$
$b = 18$
$c = 21$
$\angle A = 61$

$a^2 = 18^2 + 21^2 - 2(18)(21)\cos(61)$
$a^2 = 398.4839\ldots$
$a = \sqrt{\text{answer}}$
$a = 19.96\text{ cm}$

2 Find the missing side length

$a^2 = b^2 + c^2 - 2bc\cos(A)$
$b^2 = a^2 + c^2 - 2ac\cos(B)$
$c^2 = a^2 + b^2 - 2ab\cos(C)$



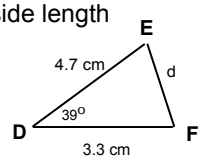
$b^2 = a^2 + c^2 - 2(a)(c)\cos(B)$

$a = 7$
$b = ?$
$c = 5$
$\angle B = 45$

$b^2 = 7^2 + 5^2 - 2(7)(5)\cos(45)$
$b^2 = 24.502\ldots$
$b = \sqrt{\text{ans}}$
$b = 4.95\text{ mm}$

3 Find the missing side length

$a^2 = b^2 + c^2 - 2bc\cos(A)$
$b^2 = a^2 + c^2 - 2ac\cos(B)$
$c^2 = a^2 + b^2 - 2ab\cos(C)$



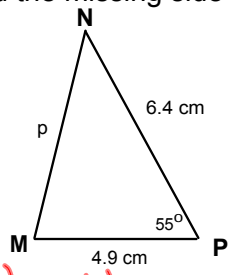
$d^2 = e^2 + f^2 - 2(e)(f)\cos(D)$

$d = ?$
$e = 3.3$
$f = 4.7$
$\angle D = 39$

$d^2 = 3.3^2 + 4.7^2 - 2(3.3)(4.7)\cos(39)$
$d^2 = 8.8729\ldots$
$d = 2.98\text{ cm}.$

4 On your own!!! Find the missing side length

$a^2 = b^2 + c^2 - 2bc\cos(A)$
$b^2 = a^2 + c^2 - 2ac\cos(B)$
$c^2 = a^2 + b^2 - 2ab\cos(C)$



$p^2 = m^2 + n^2 - 2(m)(n)\cos(P)$

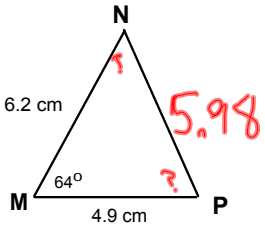
$p^2 = (6.4)^2 + (4.9)^2 - 2(6.4)(4.9)\cos(55)$

$p^2 = 28.9952\ldots$

$p = 5.38\text{ cm}$

5 Solve the triangle:

$a^2 = b^2 + c^2 - 2bc\cos(A)$
$b^2 = a^2 + c^2 - 2ac\cos(B)$
$c^2 = a^2 + b^2 - 2ab\cos(C)$



$\angle M$

$m^2 = p^2 + n^2 - 2(p)(n)\cos(M)$

$m^2 = 6.2^2 + 4.9^2 - 2(6.2)(4.9)\cos(64)$

$m = 5.98\text{ cm}$

$\angle P = 180 - 64 - 47.4$

$= 68.6°$

$\angle N$

$\frac{4.9}{\sin N} = \frac{5.98}{\sin 64}$

$4.9(\sin 64) = 5.98(\sin N)$

$\frac{4.9(\sin 64)}{5.98} = \sin N$

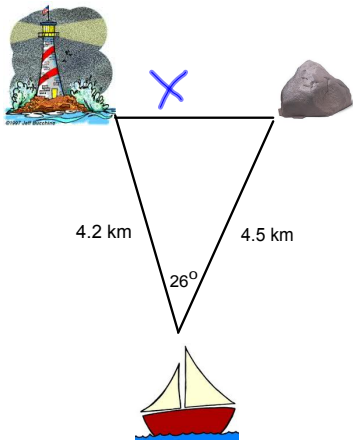
$\sin N = 0.736\ldots$

$\angle N = \sin^{-1}(\text{ans})$

$\angle N = 47.4°$

6 **Application**

A boat is sailing north through a narrow strait. Through one particularly narrow section, a lighthouse marks the western shoreline, while a buoy indicates a rock hazard directly east of the lighthouse as shown. What is the width of this section of the strait, to the nearest tenth of a kilometer.



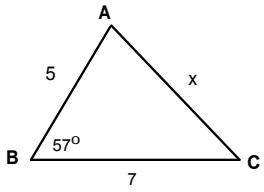
$x^2 = 4.2^2 + 4.5^2 - 2(4.2)(4.5)\cos(26$

$x = 1.98\text{ km}$

The width is 2.0 km

What did we learn today?

You can use the cosine law to find a missing side of an acute triangle if the other two sides and their contained angle are known



Cosine Law:

$a^2 = b^2 + c^2 - 2bc\cos(A)$
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Tomorrow we will:

Learn to find an unknown angle if you know three side lengths of an acute triangle

Homework:
Pg. 409 # 1,2,3,4,7,8