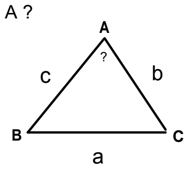
How do we find angle A ?



$a^2 = b^2 + c^2 - 2bc\cos(A)$
$b^2 = a^2 + c^2 - 2ac\cos(B)$
$c^2 = a^2 + b^2 - 2ab\cos(C)$

We can rearrange the law of cosines to solve for angle A:

$$a^2 = b^2 + c^2 - 2bc\cos(A)$$
$$a^2 - b^2 - c^2 = -2bc\cos(A)$$
$$\frac{a^2 - b^2 - c^2}{-2bc} = \cos(A)$$

Law of Cosines

1) You can use the cosine law to find a missing side of an acute triangle if the other two sides and their contained angle are known

$$a^2 = b^2 + c^2 - 2bc\cos(A)$$
$$b^2 = a^2 + c^2 - 2ac\cos(B)$$
$$c^2 = a^2 + b^2 - 2ab\cos(C)$$

2) find an unknown angle if you know three side lengths of an acute triangle

$$\cos(A) = \frac{a^2 - b^2 - c^2}{-2bc}$$
$$\cos(B) = \frac{b^2 - a^2 - c^2}{-2ac}$$
$$\cos(C) = \frac{c^2 - a^2 - b^2}{-2ab}$$

COSINE LAW

Find an angle given three sides

$$\cos(A) = \frac{a^2 - b^2 - c^2}{-2bc}$$
$$\cos(B) = \frac{b^2 - a^2 - c^2}{-2ac}$$
$$\cos(C) = \frac{c^2 - a^2 - b^2}{-2ab}$$

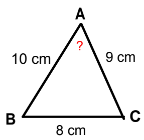
Find a side given two sides and a contained angle

$$a^2 = b^2 + c^2 - 2bc\cos(A)$$
$$b^2 = a^2 + c^2 - 2ac\cos(B)$$
$$c^2 = a^2 + b^2 - 2ab\cos(C)$$

1 Solve for the indicated angle

$\cos(A) = \frac{a^2 - b^2 - c^2}{-2bc}$
$\cos(B) = \frac{b^2 - a^2 - c^2}{-2ac}$
$\cos(C) = \frac{c^2 - a^2 - b^2}{-2ab}$

$a = 8$
$b = 9$
$c = 10$



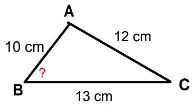
$$\cos(A) = \frac{8^2 - 9^2 - 10^2}{-2(9)(10)}$$
$$\cos(A) = \frac{-117}{-180}$$
$$\angle A = \cos^{-1}\left(\frac{-117}{-180}\right)$$
$$\angle A = 49.5^\circ$$

2 Solve for the indicated angle

$\cos(A) = \frac{a^2 - b^2 - c^2}{-2bc}$

$\cos(B) = \frac{b^2 - a^2 - c^2}{-2ac}$

$\cos(C) = \frac{c^2 - a^2 - b^2}{-2ab}$



a=13
b=12
c=10

$$\cos(B) = \frac{12^2 - 13^2 - 10^2}{-2(13)(10)}$$

$$\cos(B) = \frac{-125}{-260}$$

$$\angle B = \cos^{-1}\left(\frac{-125}{-260}\right)$$

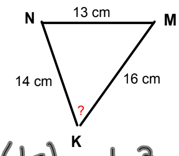
$$\angle B = 61.3^\circ$$

3 Solve for the indicated angle

$\cos(A) = \frac{a^2 - b^2 - c^2}{-2bc}$

$\cos(B) = \frac{b^2 - a^2 - c^2}{-2ac}$

$\cos(C) = \frac{c^2 - a^2 - b^2}{-2ab}$



k=13
n=16
m=14

$$\cos(K) = \frac{k^2 - n^2 - m^2}{-2(n)(m)}$$

$$\cos(K) = \frac{13^2 - 16^2 - 14^2}{-2(16)(14)}$$

$$\cos(K) = \frac{-283}{-448}$$

$$\angle K = \cos^{-1}\left(\frac{-283}{-448}\right)$$

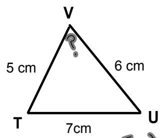
$$\angle K = 50.8^\circ$$

4 On your own!!!! Solve for angle V

$\cos(A) = \frac{a^2 - b^2 - c^2}{-2bc}$

$\cos(B) = \frac{b^2 - a^2 - c^2}{-2ac}$

$\cos(C) = \frac{c^2 - a^2 - b^2}{-2ab}$



v=7
t=6
u=5

$$\cos(V) = \frac{v^2 - u^2 - t^2}{-2ut}$$

$$\cos(V) = \frac{7^2 - 6^2 - 5^2}{-2(6)(5)}$$

$$\cos(V) = \frac{-12}{-60}$$

$$\angle V = \cos^{-1}\left(\frac{-12}{-60}\right)$$

$$\angle V = 78.5^\circ$$

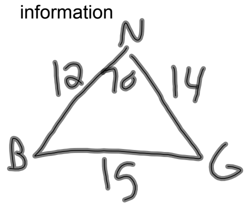
5 In acute △ NBG , n = 15 m, b = 14 m, g = 12 m

$\cos(A) = \frac{a^2 - b^2 - c^2}{-2bc}$

$\cos(B) = \frac{b^2 - a^2 - c^2}{-2ac}$

$\cos(C) = \frac{c^2 - a^2 - b^2}{-2ab}$

a) sketch the triangle and label it with the given information



$$\cos(N) = \frac{n^2 - b^2 - m^2}{-2(b)(m)}$$

$$\cos(N) = \frac{15^2 - 14^2 - 12^2}{-2(14)(12)}$$

$$\angle N = 70^\circ$$

b) Solve the triangle

$\angle B$

$$\frac{15}{\sin 70} = \frac{14}{\sin B}$$

$$15(\sin B) = 14(\sin 70)$$

$$\sin B = \frac{14(\sin 70)}{15}$$

$$\sin B = 0.877....$$

$$\angle B = 61°$$

$\angle G$

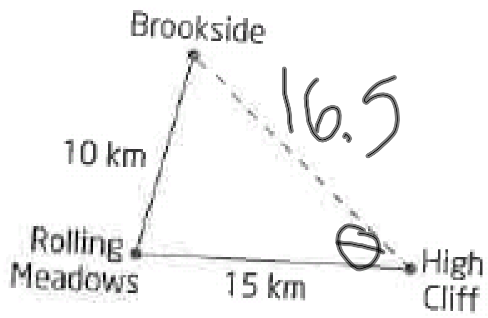
$$= 180 - 61 - 70$$

$$= 49°$$

6 Three towns are connected by two roads as shown. A third road is planned that will connect Brookside with High Cliff, which are 16.5 km apart. Find the angle between the new road and the existing road from:

a) High Cliff to Rolling Meadows

~~**b)** Brookside to Rolling Meadows~~



a)

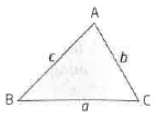
$$\cos(\theta) = \frac{-10^2 - 16.5^2 - 15^2}{-2(16.5)(15)}$$

$$\angle \theta = 36.6°$$

b)

Homework: 8.3 Worksheet

Key Concepts



- You can rearrange the cosine law to find an angle if you know three side lengths of an acute △ABC.

  For example, to find the measure of ∠B, rearrange the appropriate form of the cosine law.

  $$b^2 = a^2 + c^2 - 2ac(\cos B) \Rightarrow \cos B = \frac{b^2 - a^2 - c^2}{-2ac}$$

- Once you have found one angle, you can apply the sine law to find a second angle.
- When applying trigonometry to solve problems involving acute triangles, there is often more than one valid strategy.