2.1/2.2 Restricting, Simplifying, Multiplying, and Dividing Rational Expressions

Lesson Outline:

Part 1: Stating restrictions

Part 2: Simplifying rational expressions

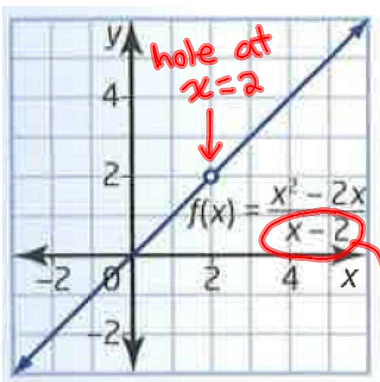
Part 3: Multiplying rational expressions

Part 4: Dividing rational expressions

What is a rational expression?

Rational expression: the quotient of two polynomials, $\frac{p(x)}{q(x)}$, where $q(x) \neq 0$.

Example of a graph of a rational expression:



The open circle is used to represent a hole in the graph. This corresponds to any restrictions on the variable (denominator can't be 0).

$x - 2 \neq 0$

$x \neq 2$

Stating Restrictions

Note: rational expressions must be checked for restrictions by determining where the denominator is equal to **zero**. These restrictions must be stated when the expression is simplified.

bottom of a fraction can NOT = 0.

Example 1: State the restrictions for the following rational expressions

a) $\frac{x+2}{x-2}$

$x-2 \neq 0$

$x \neq 2$

$\boxed{x \neq 2}$

b) $\frac{x+2}{(x-3)(x+4)}$

$x-3 \neq 0$

$x \neq 3$

$x+4 \neq 0$

$x \neq -4$

$\boxed{x \neq 3, -4}$

c) $\frac{5}{x(x+3)}$

$x \neq 0$

$x+3 \neq 0$

$x \neq -3$

$\boxed{x \neq 0, -3}$

Rule: We can cancel out ONLY when multiplying fractions

You can cancel factors.

eg. $\dfrac{\cancel{x}(x+1)}{\cancel{x}} = x+1$

eg. $\dfrac{3\cancel{6x}(x+3)}{2\cancel{4x}} = \dfrac{3x+9}{2}$

These are examples of when cancelling is allowed.

Rule: We can **NOT** cancel out when adding or subtracting fractions

e.g. $\dfrac{\cancel{x}+8}{\cancel{x}}$ NO! Don't do this!

e.g. $\dfrac{2x+3}{4x}$ No cancelling!

Simplifying Rational Expressions

Example 2: Simplifying each expression and determine any restrictions on the variable.

a) $\dfrac{3x^{\cancel{2}\,1}}{y\cancel{x}}$, $y \neq 0$
$x \neq 0$

$= \dfrac{3x}{y}$, $y \neq 0$
$x \neq 0$

b) $\dfrac{x-3}{x^2+3x-18}$

Note: *factor where possible and then state restrictions before cancelling factors.*

$= \dfrac{\cancel{x-3}}{(x+6)\cancel{(x-3)}}$, $x \neq -6, 3$

$= \dfrac{1}{x+6}$, $x \neq -6, 3$

c)

$$\frac{x^2 + 10x + 21}{x + 3} \quad ; x \neq -3$$

$$= \frac{(x+7)\cancel{(x+3)}}{\cancel{x+3}} \quad ; x \neq -3$$

$$= x + 7 \quad ; x \neq -3$$

d)

$$\frac{x+1}{x^2 + 3x + 2}$$

$$= \frac{\cancel{x+1}}{(x+2)\cancel{(x+1)}} \quad ; x \neq -2, -1$$

$$= \frac{1}{x+2}$$

e)

$$\frac{x^2-9}{x^2+7x+12}$$ ← difference of squares: $a^2-b^2=(a-b)(a+b)$

$$=\frac{(x-3)\cancel{(x+3)}}{(x+4)\cancel{(x+3)}} \; ; x \neq -4, -3$$

$$=\frac{x-3}{x+4} \; ; x \neq -4, -3$$

f)

$$\frac{6x^2-7x-5 \, *}{3x^2+x-10 \, *}$$

Factor numerator

$6x^2-10x+3x-5$

$=(6x^2-10x)+(3x-5)$

$=2x(3x-5)+1(3x-5)$

$=(2x+1)(3x-5)$

Factor denominator

$3x^2+x-10$

$=3x^2+6x-5x-10$

$=(3x^2+6x)+(-5x-10)$

$=3x(x+2)-5(x+2)$

$=(x+2)(3x-5)$

$$=\frac{(2x+1)\cancel{(3x-5)}}{(x+2)\cancel{(3x-5)}} \; ; x \neq -2, \frac{5}{3}$$

$$=\frac{2x+1}{x+2} \; ; x \neq -2, \frac{5}{3}$$

Multiplying Rational Expressions

1. factor where possible
2. cancel common factors
3. multiply numerators and denominators
4. state restrictions (throughout process)

a)

$$\frac{{}^{2}\cancel{4}x^{\cancel{2}\,1}}{{}^{1}\cancel{3}\cancel{x}} \bullet \frac{{}^{4}\cancel{12}x^{\cancel{3}\,2}}{{}^{1}\cancel{2}\cancel{x}} \quad ; x \neq 0$$

$$= 8x^3 \quad ; x \neq 0$$

b)

$$\frac{4x+24}{x^2+8x} \bullet \frac{12x^2}{3x+18}$$

$$= \frac{4\cancel{(x+6)}}{\cancel{x}(x+8)} \cdot \frac{{}^{4}\cancel{12}x^{\cancel{2}\,1}}{{}^{1}\cancel{3}\cancel{(x+6)}} \quad ; x \neq 0, -8, -6$$

$$= \frac{16x}{x+8} \quad ; x \neq 0, -8, -6$$

c)

$$\frac{x+1}{2x} \bullet \frac{3x}{x^2+4x+3} \quad ; x \neq 0$$

$$= \frac{\cancel{x+1}}{2\cancel{x}} \cdot \frac{3\cancel{x}}{\cancel{(x+1)}(x+3)} \quad ; x \neq 0, -1, -3$$

$$= \frac{3}{2(x+3)} \quad ; x \neq 0, -1, -3$$

d)

$$* \frac{5x^2-13x+8}{x-7} \bullet \frac{1}{5x-8}$$

factor
$= 5x^2 - 8x - 5x + 8$
$= (5x^2 - 8x) + (-5x + 8)$
$= x(5x-8) - 1(5x-8)$
$= (5x-8)(x-1)$

$$= \frac{\cancel{(5x-8)}(x-1)}{x-7} \cdot \frac{1}{\cancel{5x-8}} \quad ; x \neq 7, \frac{8}{5}$$

$$= \frac{x-1}{x-7} \quad ; x \neq 7, \frac{8}{5}$$

Dividing Rational Expressions

1. flip second fraction and change to multiplication
2. factor where possible
3. cancel common factors
4. multiply numerators and denominators
5. state restrictions (throughout process)

a)

$$\frac{10ab^2}{4a} \div \frac{15a^2}{12b^2}$$

no cross cancelling until after second fraction has been flipped

$$= \frac{10ab^2}{4a} \times \frac{12b^2}{15a^2} \quad ; \quad a \neq 0, \; b \neq 0$$

← because 'b' was in the denominator of the original expression

$$= \frac{\overset{2}{\cancel{120}}\,ab^4}{\cancel{60}\,a^{\cancel{3}\,2}} \quad ; \quad a \neq 0, \; b \neq 0$$

$$= \frac{2b^4}{a^2} \quad ; \quad a \neq 0, \; b \neq 0$$

b)

$$\frac{a^2 + 2a}{3a} \div \frac{5a^2 + 10a}{20a^2}$$

$$= \frac{a^2+2a}{3a} \times \frac{20a^2}{5a^2+10a} \quad ; \quad a \neq 0$$

$$= \frac{\cancel{a}\,\cancel{(a+2)}}{3\cancel{a}} \times \frac{\overset{4}{\cancel{20}}\,a^{\cancel{2}\,1}}{\cancel{5a}\,\cancel{(a+2)}} \quad ; \quad a \neq 0, -2$$

$$= \frac{4a}{3} \quad ; \quad a \neq 0, -2$$

c)

$$\frac{2x^2-8x}{x^2-3x-10} \div \frac{4x^2}{x^2-9x+20}$$

$$= \frac{2x^2-8x}{x^2-3x-10} \times \frac{x^2-9x+20}{4x^2}$$

$$= \frac{\overset{1}{\cancel{2}}\cancel{x}(x-4)}{\cancel{(x-5)}(x+2)} \times \frac{\cancel{(x-5)}(x-4)}{\underset{2}{\cancel{4}}x^{\cancel{2}\,1}}$$

$$= \frac{(x-4)^2}{2x(x+2)} \quad ; x \neq -2, 0, 4, 5$$

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