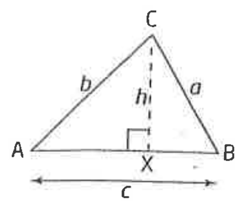
Proof of Sine Law - $\frac{a}{\text{Sin A}} = \frac{b}{\text{Sin B}} = \frac{c}{\text{Sin C}}$

The relationship you investigated is called the **sine law**. To show why the sine law holds true, draw an acute triangle and add an altitude, h, from one of the vertices.



sine law

- the relationship between the sides and their opposite angles in any acute △ABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

The altitude splits △ABC into two smaller right triangles, △AXC and △BXC.

Find an expression for h.

Focus on △AXC:

$$\sin A = \frac{h}{b}$$ Multiply both sides by b.

$$b(\sin A) = h$$

Focus on △BXC:

$$\sin B = \frac{h}{a}$$ Multiply both sides by a.

$$a(\sin B) = h$$

Set the two expressions for h equal.

$$b(\sin A) = a(\sin B)$$

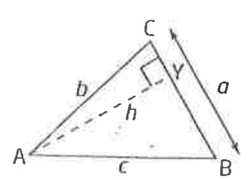
$$\frac{b(\cancel{\sin A})}{\cancel{\sin A}} = \frac{a(\sin B)}{\sin A}$$ Divide both sides by sin A.

$$b = \frac{a(\sin B)}{\sin A}$$

$$\frac{b}{\sin B} = \frac{a(\cancel{\sin B})}{\sin A(\cancel{\sin B})}$$ Divide both sides by sin B.

$$\frac{b}{\sin B} = \frac{a}{\sin A}$$

This process can be repeated using a different altitude. Combining the results gives the sine law.



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Even though there are three parts to this equation, you only use two parts at a time. The choice of which two to use depends on what information is given.

You can apply the sine law to find an unknown side length of an acute triangle if you know two angles and one of the side lengths.

Proof of Cosine Law -

$a^2 = b^2 + c^2 - 2bc \cos(A)$
$b^2 = a^2 + c^2 - 2ac \cos(B)$
$c^2 = a^2 + b^2 - 2ab \cos(C)$

cosine law

- the relationship between the cosine of an angle and the lengths of the three sides in any acute △ABC:
  $a^2 = b^2 + c^2 - 2bc(\cos A)$
  $b^2 = c^2 + a^2 - 2ca(\cos B)$
  $c^2 = a^2 + b^2 - 2ab(\cos C)$

The **cosine law** relates the cosine of an angle to three side lengths of an acute triangle. To derive cosine law, draw a triangle and add an altitude, *i* from one of the vertices.

The altitude splits △ABC into two smaller right triangles, △ADC and △BDC.

Let AD = x.
Then, BD = $c - x$.

Focus on △ADC:
From the Pythagorean theorem, $b^2 = x^2 + h^2$.
Also, the cosine ratio gives

$$\frac{x}{b} = \cos A$$

$$x = b(\cos A)$$

Focus on △BDC:
Write an equation using the Pythagorean theorem.

$a^2 = h^2 + (c - x)^2$
$= h^2 + c^2 - 2cx + x^2$ — Expand the binomial.
$= x^2 + h^2 + c^2 - 2cx$ — Rearrange the terms.
$= b^2 + c^2 - 2c[b(\cos A)]$ — Substitute b^2 for $x^2 + h^2$
$= b^2 + c^2 - 2cb(\cos A)$
$= b^2 + c^2 - 2bc(\cos A)$

This equation allows you to find the side length a i lengths b and c and the measure of ∠A.

You can derive similar equations for the other side the results gives the three forms of the cosine law.

$a^2 = b^2 + c^2 - 2bc(\cos A)$
$b^2 = c^2 + a^2 - 2ca(\cos B)$
$c^2 = a^2 + b^2 - 2ab(\cos C)$