Grade 10 Math in 1 Hour

Unit 1 – Linear Systems

1) Solve the following linear system in 3 ways: GRAPHING, SUBSTITUTION, and ELIMINATION

① $x - y = 5$
② $3x + y = 3$

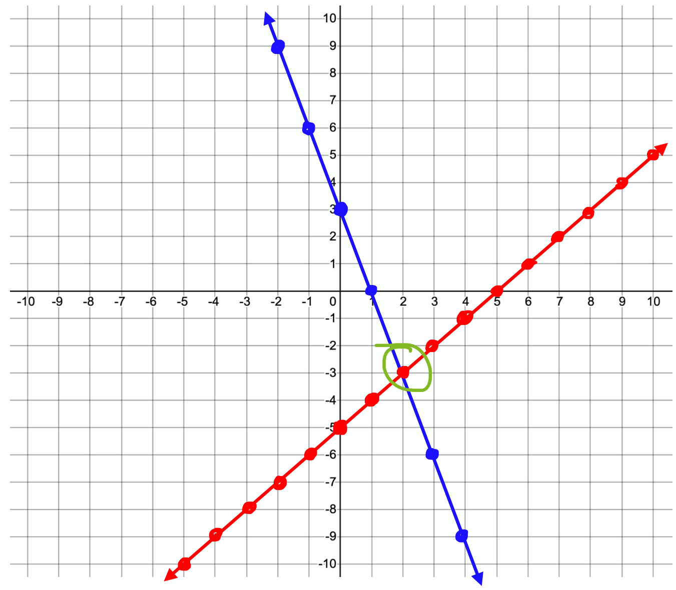
① $x - y = 5$
$x - 5 = y$

② $3x + y = 3$
$y = -3x + 3$

Solution: $(2, -3)$
or $x = 2, y = -3$

Solving by GRAPHING

1) Re-arrange both equations into the form $y = mx + b$
2) Use the slope (m) and the y-int (b) to graph both lines.
3) The point of intersection is the solution to the system.



Solving by ELIMINATION

1) Re-arrange both equations into the form $x + y = \#$
2) Make the coefficients of either the x or y variables have the same absolute value by multiplying either or both equations by a constant.
3) Add or Subtract the equations to eliminate a variable
4) Solve for the remaining variable
5) Plug answer back in to either original equation to solve for the OTHER variable.

① $x - y = 5$
② $3x + y = 3$ +

①+②
$4x = 8$
$x = 2$

sub $x=2$ in to ① or ②

$2 - y = 5$
$y = -3$

solution: $(2, -3)$

Solving by SUBSTITUTION

1) Re-arrange either equation to isolate a variable $(x$ or $y)$
2) Substitute what the isolated variable is equal to into the OTHER equation
3) Solve the new equation for the variable
4) Plug answer back in to either original equation to solve for the OTHER variable.

① $x - y = 5$
$x = 5 + y$

② $3x + y = 3$
$3(5+y) + y = 3$
$15 + 3y + y = 3$
$15 + 4y = 3$
$4y = -12$
$y = -3$

$x = 5 + (-3)$
$x = 2$

solution: $(2, -3)$

2) Solve the following linear system using the method of elimination

① $4x + 3y = 13$
② $5x - 4y = -7$

> **Tip:**
>
> If variables have the SAME sign, SUBTRACT to eliminate
>
> If variables have the OPPOSITE sign, ADD to eliminate

$5\times$① $20x + 15y = 65$
$4\times$② $20x - 16y = -28$ −

$31y = 93$
$y = 3$

sub $y=3$ into ① or ②

$4x + 3(3) = 13$
$4x + 9 = 13$
$4x = 4$
$x = 1$

$\boxed{\text{solution: } (1, 3)}$

3) The Sports Shop sells Adidas running shoes for \$82 a pair and Air Jensen basketball shoes for \$95 a pair. One day, the Sports Shop sells a combined 75 pairs of Adidas and Air Jensen shoes totaling \$6241 in sales. How many pairs of each shoes were sold?

> **Tip:**
>
> Remember that these questions are designed so that there are 2 variables that need to be solved for.
>
> They will ALWAYS give you enough information to make 2 equations involving these 2 variables.

x = # of adidad
y = # of AJ

① $x + y = 75$
② $82x + 95y = 6241$
$82\times$① $82x + 82y = 6150$ −

$13y = 91$
$y = 7$

sub $y=7$ into ① or ②

$x + 7 = 75$
$x = 68$

∴ they sold 68 pairs of Adidas & 7 pairs of Air Jensen's.

4) A blue spruce tree grows an average of 15 cm per year. An eastern hemlock grows an average of 10 cm per year. When they were planted, a blue spruce was 120 cm tall and an eastern hemlock was 180 cm tall. How many years after planting will the trees reach the same height? How tall will that be?

t = # of years
h = height

① Blue spruce: $h = 15t + 120$
② E. hemlock: $h = 10t + 180$

$10t + 180 = 15t + 120$
$60 = 5t$
$t = 12$ years

sub $t=12$ into ① or ②

$h = 10(12) + 180$
$h = 120 + 180$
$h = 300$ cm

∴ After 12 years both trees will be 300 cm tall.

5) Calculate the midpoint and distance between the following points

$A(5,-3)$ and $B(-1,5)$

Formulas Needed

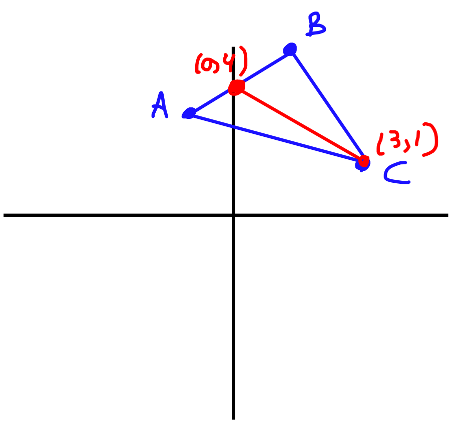
Midpoint: $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Distance: $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$$\text{Midpoint} = \left(\frac{5+(-1)}{2}, \frac{-3+5}{2}\right)$$
$$= \left(\frac{4}{2}, \frac{2}{2}\right)$$
$$= (2, 1)$$

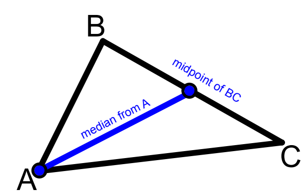
$$\text{Distance} = \sqrt{(-1-5)^2+[5-(-3)]^2}$$
$$= \sqrt{(-6)^2+(8)^2}$$
$$= \sqrt{36+64}$$
$$= \sqrt{100}$$
$$= 10 \text{ units}$$

6) Draw the triangle with vertices A(-1,3) B(1,5) and C(3,1). Draw the median from vertex C to AB. Then find the equation of this median.



Median

A median of a triangle is the line segment that joins a vertex to the midpoint of the opposite side.



$$\text{mid of } AB = \left(\frac{-1+1}{2}, \frac{3+5}{2}\right)$$
$$= (0, 4)$$

$$\text{slope of median} = m = \frac{1-4}{3-0} = \frac{-3}{3} = -1$$

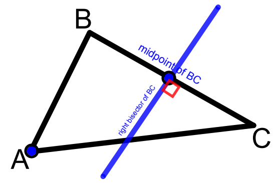
y-int of median:
$$y = mx+b$$
$$4 = -1(0)+b$$
$$b = 4$$

$$\boxed{\text{Eq}^n \text{ of median: } y = -x+4}$$

7) Determine the equation for the right bisector of the line segment with endpoints A(-2,-4) and B(8,6)

Perpendicular Bisector

A line that is perpendicular to a side of a triangle AND passes through its midpoint.



Slope of AB $= m = \dfrac{6-(-4)}{8-(-2)} = \dfrac{10}{10} = 1$

slope of Right Bisector $= \perp m = -1$

midpoint of AB $= \left(\dfrac{-2+8}{2}, \dfrac{-4+6}{2}\right)$

$= (3,1)$

y-int of Right bisector: $y = mx+b$

$1 = -1(3)+b$

$b = 4$

$\boxed{\text{Eq}^n \text{ of Right bisector: } y = -x+4}$

8) Classify the triangle with vertices of D(–4, –2), E(–2, 6), and F(6, –4) as either scalene, isosceles, or equilateral. Also state if it has a right angle.

Formula Needed

Distance: $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Tip:

If a triangle has a RIGHT angle, Pythagorean Theorem will hold true.

$a^2 + b^2 = c^2$

Distance calculations:

$DE = \sqrt{[-2-(-4)]^2 + [6-(-2)]^2} = \sqrt{4+64} = \sqrt{68}$

$EF = \sqrt{[6-(-2)]^2 + (-4-6)^2} = \sqrt{64+100} = \sqrt{164}$

$DF = \sqrt{[6-(-4)]^2 + [-4-(-2)]^2} = \sqrt{100+4} = \sqrt{104}$

Check for right angle:

$\sqrt{68}^2 + \sqrt{104}^2 \stackrel{?}{=} \sqrt{164}^2$

$68 + 104 \stackrel{?}{=} 164$

$172 \neq 164$

$\therefore$ scalene with no right angle.

9) For the circle that is centered at the origin and passes through the point $(-3,4)$

a) Find the equation of the circle

Formulas Needed

Equation of ANY Circle:

$$(x-h)^2+(y-k)^2=r^2$$

Equation of Circle Centered at ORIGIN:

$$x^2+y^2=r^2$$

$x^2+y^2=r^2$

$(-3)^2+(4)^2=r^2$

$9+16=r^2$

$r^2=25$

Eq^n: $x^2+y^2=25$

b) Does the point (5,2) lie on the circle, inside of it, or outside of it?

Tip:

If point (x,y) is **ON** the circle $\rightarrow x^2+y^2=r^2$

If point (x,y) is **OUTSIDE** the circle $\rightarrow x^2+y^2>r^2$

If point (x,y) is **INSIDE** the circle $\rightarrow x^2+y^2<r^2$

$x^2+y^2=25$

$5^2+2^2>25$

$29>25$

$\therefore$ Outside of the circle.

10) What is the shortest distance from the point at (-3,5) to the line $y = \frac{1}{4}x + 10$

1) $\perp m = -4$

$y = mx + b$

$5 = -4(-3) + b$

$b = -7$

$y = -4x - 7$

2) $y = \frac{1}{4}x + 10$

$y = -4x - 7$

$-4x - 7 = \frac{1}{4}x + 10$

$-16x - 28 = x + 40$

$-68 = 17x$

$x = -4$

$y = -4(-4) - 7$

$y = 9$

POI is $(-4, 9)$

3) Distance from $(-4,9)$ to $(-3,5)$

$$\text{Distance} = \sqrt{[-3-(-4)]^2 + (5-9)^2}$$

$$= \sqrt{1 + 16}$$

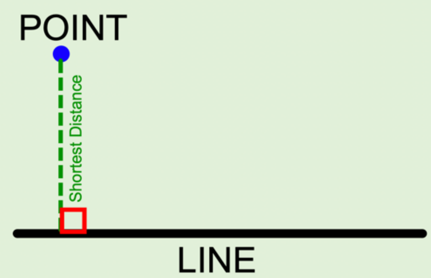
$$= \sqrt{17} \text{ units}$$

STEPS FOR FINDING SHORTEST DISTANCE

1) Find equation of a line that is perpendicular to the original line and passes through the given point.
2) Find where that line intersects the original line.
3) Find the distance from the original point to the point where the lines intersect.

Tip:

The shortest distance from a POINT to a LINE is always the in a direction that is PERPENDICULAR to the line.



Unit 2 – Quadratics

Standard Form: $y = ax^2 + bx + c$

y-int: $(0, c)$

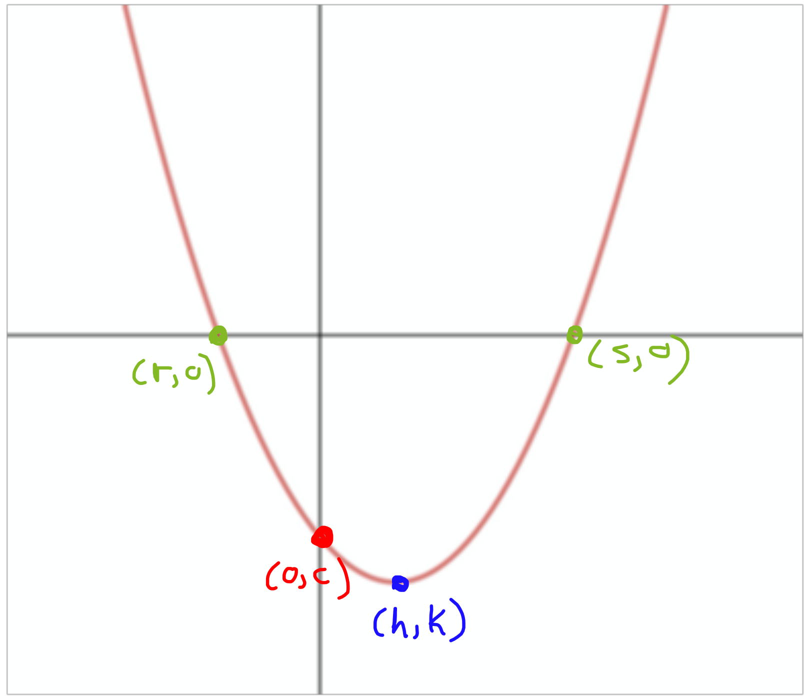
$a > 0$; opens up
$a < 0$; opens down

Vertex Form: $y = a(x - h)^2 + k$

vertex: (h, k)

Factored Form: $y = a(x - r)(x - s)$

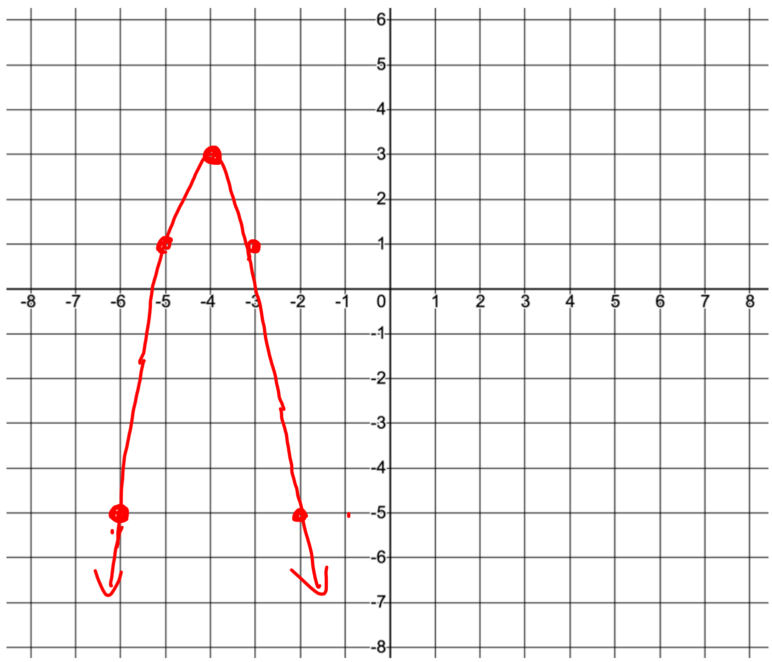
x-int: $(r, 0)$ and $(s, 0)$



11) For the quadratic $y = -2(x+4)^2 + 3$, complete the table of information, and then graph the function using its properties and a table of values.

Property	$y = -2(x+4)^2 + 3$
Vertex	$(-4, 3)$
axis of symmetry	$x = -4$
stretch or compression	vertical stretch BAF 2
direction of opening	Down
values that x may take	$x \in \mathbb{R}$
values that y may take	$y \leq 3$

x	y
-6	-5
-5	1
-4	3
-3	1
-2	-5



12) Find vertex form equation of the following quadratic.

$y = a(x-h)^2 + k$

$-4 = a(0-2)^2 + (-6)$

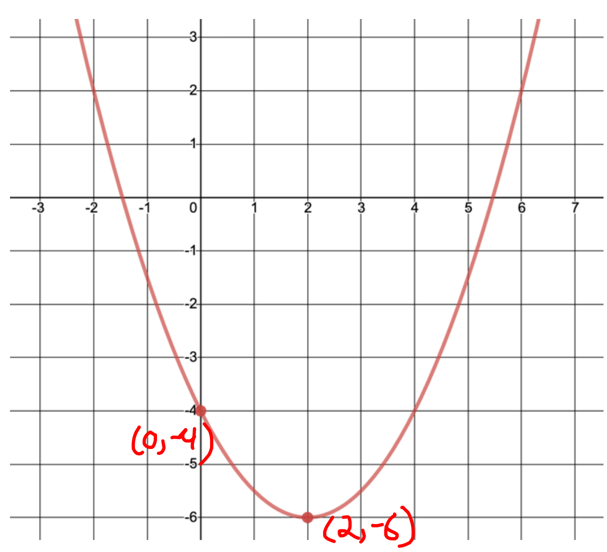
$-4 = a(4) - 6$

$-4 = 4a - 6$

$2 = 4a$

$a = \frac{1}{2}$

$\boxed{y = \frac{1}{2}(x-2)^2 - 6}$



13) Describe the transformations from $y = x^2$ to $y = -\frac{1}{3}(x+5)^2 - 4$

Shift left 5

shift down 4

vertical compression bofo $\frac{1}{3}$

vertical reflection

14) For the quadratic $y = -4(x-2)(x-4)$, state the x-intercepts, vertex and axis of symmetry. Then use this information to graph it.

x-int: $(2,0)$ and $(4,0)$

x-vertex $= \frac{2+4}{2} = \frac{6}{2} = 3$

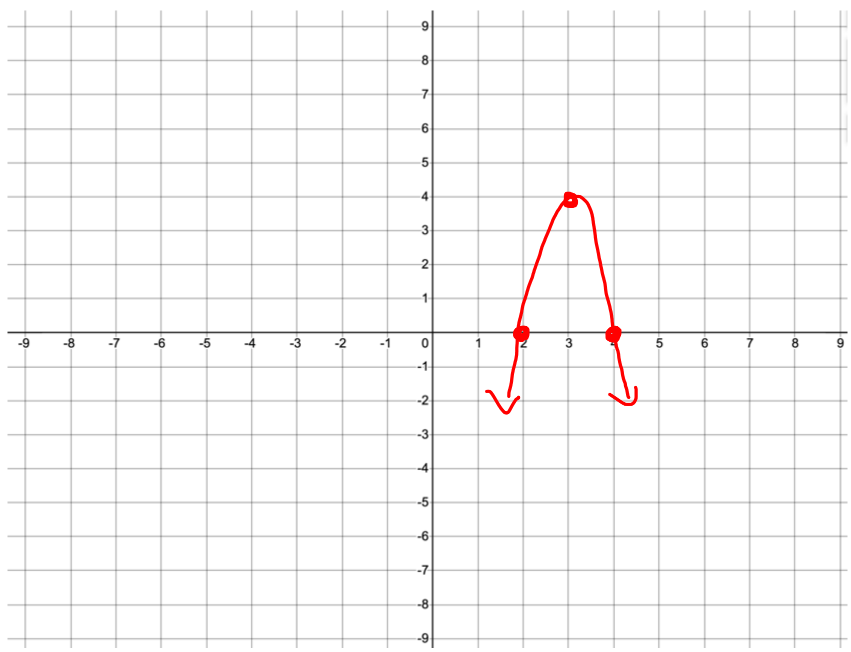
y-vertex $= -4(3-2)(3-4)$

$= -4(1)(-1)$

$= 4$

vertex $= (3,4)$

aos: $x = 3$



r s x y

15) A parabola has x-intercepts -8 and 2, and passes through the point (0, -8). Determine the equation of this parabola in the form $y = a(x-r)(x-s)$.

$-8 = a(0+8)(0-2)$

$-8 = a(8)(-2)$

$-8 = -16a$

$a = \frac{1}{2}$

$\boxed{y = \frac{1}{2}(x+8)(x-2)}$

Standard form to factored form

$$ax^2 + bx + c \rightarrow a(x - r)(x - s)$$

16) Factor each of the following

Tip:

ALWAYS check for a common factor first!

a) $y = x^2 + 10x + 24$

p: 24
s: 10
6 and 4

$y = (x+6)(x+4)$

b) $y = x^2 + x - 12$

p: -12
s: 1
4 and -3

$y = (x+4)(x-3)$

c) $y = 2x^2 + 22x + 48$

$y = 2(x^2 + 11x + 24)$

p: 24
s: 11
8 and 3

$y = 2(x+8)(x+3)$

d) $y = 2x^2 + 7x - 15$

p: -30
s: 7
10 and -3

$y = 2x^2 + 10x - 3x - 15$

$y = 2x(x+5) - 3(x+5)$

$y = (x+5)(2x-3)$

e) $y = 3x^2 + 23x - 8$

p: -24
s: 23
24 and -1

$y = 3x^2 + 24x - 1x - 8$

$y = 3x(x+8) - 1(x+8)$

$y = (x+8)(3x-1)$

f) $y = x^2 - 16$

$y = (x)^2 - (4)^2$

$y = (x-4)(x+4)$

g) $y = 4x^2 - 25$

$y = (2x)^2 - (5)^2$

$y = (2x-5)(2x+5)$

h) $y = x^2 + 6x + 9$

p: 9
s: 6
3 and 3

$y = (x+3)(x+3)$

$y = (x+3)^2$

Factored to standard

$$a(x-r)(x-s) \rightarrow ax^2+bx+c$$

17) Expand each of the following into standard form

a) $(4x-1)(x+7)$

$= 4x^2+28x-x-7$

$= 4x^2+27x-7$

b) $(x-5)^2$

$= (x-5)(x-5)$

$= x^2-5x-5x+25$

$= x^2-10x+25$

Standard form to vertex form

$$ax^2+bx+c \rightarrow a(x-h)^2+k$$

18) Convert each of the following into vertex form by completing the square. State the vertex and whether it is a max or min point.

a) $x^2+6x+11$

$y=(x^2+6x)+11$

$y=(x^2+6x+9-9)+11$

$y=(x^2+6x+9)-9+11$

$y=(x+3)^2+2$

vertex: $(-3,2)$

vertex is a min

b) $3x^2+24x-17$

$y=(3x^2+24x)-17$

$y=3(x^2+8x)-17$

$y=3(x^2+8x+16-16)-17$

$y=3(x^2+8x+16)-48-17$

$y=3(x+4)^2-65$

vertex: $(-4,-65)$

vertex is a min

Completing the Square Steps

$ax^2+bx+c \rightarrow a(x-h)^2+k$

1) Put brackets around the first 2 terms
2) Factor out the constant in front of the x^2 term
3) Look at the last term in the brackets, divide it by 2 and then square it
4) Add AND subtract that term behind the last term in the brackets
5) Move the negative term outside the brackets by multiplying it by the $'a'$ value
6) Simplify the terms outside the brackets
7) Factor the perfect square trinomial

$a^2+2ab+b^2=(a+b)^2$

19) Solve each of the following quadratic equations using the most appropriate method.

Quadratic Formula:
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Difference of Squares:
$a^2 - b^2 = (a - b)(a + b)$

Tip:

Always try to solve by factoring BEFORE resorting to quadratic formula.

Tip:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Quadratic formula will give you:

2 solutions if $b^2 - 4ac > 0$

1 solutions if $b^2 - 4ac = 0$

0 solutions if $b^2 - 4ac < 0$

a) $0 = x^2 - 36$

$0 = (x-6)(x+6)$

$x-6=0$ $\quad$ $x+6=0$

$\boxed{x=6}$ $\quad$ $\boxed{x=-6}$

b) $x^2 + 4x = 21$

$x^2+4x-21=0$

$(x+7)(x-3)=0$

$x+7=0$ $\quad$ $x-3=0$

$\boxed{x=-7}$ $\quad$ $\boxed{x=3}$

c) $-x^2 + 5x + 6 = 0$

$0 = x^2-5x-6$

$0 = (x-6)(x+1)$

$x-6=0$ $\quad$ $x+1=0$

$\boxed{x=6}$ $\quad$ $\boxed{x=-1}$

d) $5x^2 - 19x = 4$

$5x^2-19x-4=0$

$5x^2-20x+1x-4=0$

$5x(x-4)+1(x-4)=0$

$(x-4)(5x+1)=0$

$x-4=0$ $\quad$ $5x+1=0$

$\boxed{x=4}$ $\quad$ $\boxed{x=\frac{-1}{5}}$

e) $x^2 + 7x + 5 = 0$

$$x = \frac{-7 \pm \sqrt{(7)^2-4(1)(5)}}{2(1)}$$

$$x = \frac{-7 \pm \sqrt{29}}{2}$$

$x = \frac{-7+\sqrt{29}}{2}$ $\quad$ $x = \frac{-7-\sqrt{29}}{2}$

$\boxed{x=-0.81}$ $\quad$ $\boxed{x=-6.19}$

f) $0 = 2x^2 + 4x + 7$

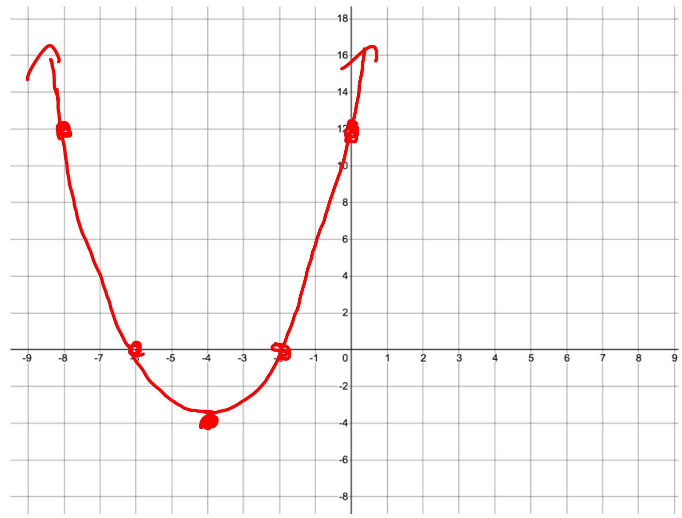
$$x = \frac{-4 \pm \sqrt{(4)^2-4(2)(7)}}{2(2)}$$

$$x = \frac{-4 \pm \sqrt{-40}}{4}$$

$\therefore$ no Real solutions

20) Sketch a graph and label all key properties of $y = x^2 + 8x + 12$

$y = (x+2)(x+6)$



x-int

$0 = (x+2)(x+6)$

$x+2=0$ $\quad$ $x+6=0$

$x=-2$ $\quad$ $x=-6$

$(-2,0)$ $\quad$ $(-6,0)$

vertex

$x\text{-vertex} = \dfrac{-2+(-6)}{2}$

$= -4$

$y\text{-vertex} = (-4)^2+8(-4)+12$

$= -4$

$(-4,-4)$

y-int $\quad y = 0^2+8(0)+12$

$y=12$ $\quad$ $(0,12)$

21) An object in launched upward at 64 feet per second (ft/s) from a platform 80 feet high. The equation for the object's height in ft based on time in seconds is given by $h = -16x^2 + 64x + 80$

a) When does the object land on the ground?

$0 = -16(x^2-4x-5)$

$0 = -16(x-5)(x+1)$

$x-5=0$ $\quad$ $x+1=0$

$x=5$ $\quad$ ~~$x=-1$~~

5 seconds

Tip:

These questions are always going to ask you for the x-intercepts and vertex.

Ground → x-int

Max or Min → vertex

b) What is the max height of the object?

$x\text{-vertex} = \dfrac{5+(-1)}{2} = 2$ seconds

$y\text{-vertex} = -16(2)^2+64(2)+80 = 144$ ft

The max height is 144 feet

c) When is the object 100 feet off the ground?

$100 = -16x^2+64x+80$

$0 = -16x^2+64x-20$

$0 = -4(4x^2-16x+5)$

$0 = 4x^2-16x+5$

$x = \dfrac{16 \pm \sqrt{(-16)^2-4(4)(5)}}{2(4)}$

$x = \dfrac{16 \pm \sqrt{176}}{8}$

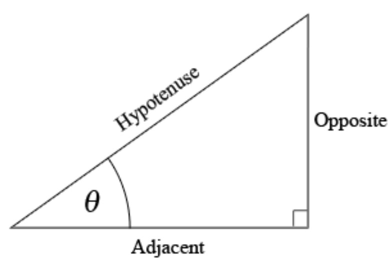
$\boxed{x = 3.66 \text{ sec.}}$ $\quad$ $\boxed{x = 0.34 \text{ sec.}}$

Unit 3: Trigonometry

Rule	When to Use It	
Pythagorean Theorem $a^2 + b^2 = c^2$	Right Triangle Know: 2 sides Want: 3rd side	
$S\frac{O}{H}C\frac{A}{H}T\frac{O}{a}$	Right Triangle Know: 2 sides Want: Angle (use inverse ratio)	Right Triangle Know: 1 side, 1 angle Want: Side
Sine Law $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$	Oblique Triangle (no right angle) Know: 2 sides and opposite angle Want: Angle	Oblique Triangle (no right angle) Know: 1 side and all angles Want: Side
Cosine Law $a^2 = b^2 + c^2 - 2bc(\cos A)$ $\cos A = \frac{a^2 - b^2 - c^2}{-2bc}$	Oblique Triangle Know: 2 sides and contained angle Want: 3rd side (use top formula)	Oblique Triangle Know: All 3 sides Want: Angle (use bottom formula)

What is $S\frac{O}{H}C\frac{A}{H}T\frac{O}{A}$?

If we know a right-angle triangle has an angle of θ, all other right-angle triangles with an angle of θ are **similar** and therefore have equivalent ratios of corresponding sides. The three primary ratios are shown in the diagram to the right.



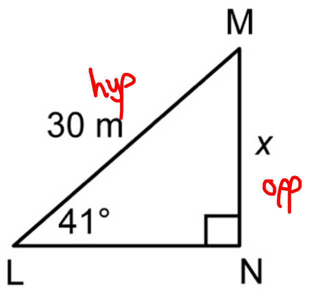
$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

22) Find the indicated missing side or angle in each triangle:

a)

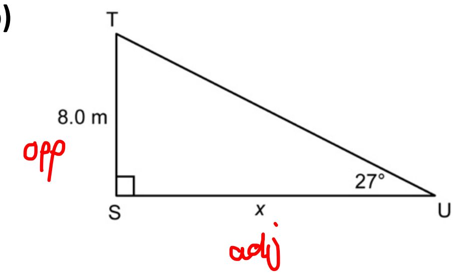


$\sin(41°) = \frac{x}{30}$

$x = 30 \sin(41°)$

$x \simeq 19.68$ m

b)

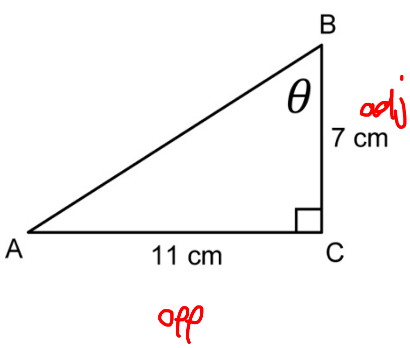


$\tan(27°) = \frac{8}{x}$

$x = \frac{8}{\tan(27°)}$

$x \simeq 15.7$ m

c)

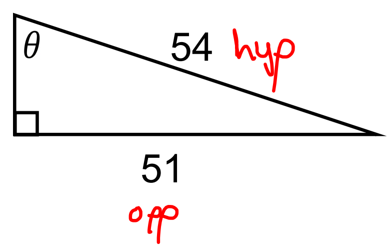


$\tan\theta = \frac{11}{7}$

$\theta = \tan^{-1}\left(\frac{11}{7}\right)$

$\theta \simeq 57.53°$

d)

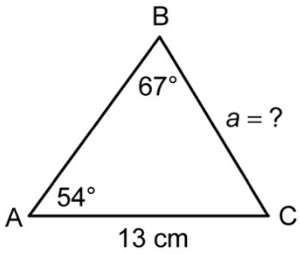


$\sin\theta = \frac{51}{54}$

$\theta = \sin^{-1}\left(\frac{51}{54}\right)$

$\theta \simeq 70.81°$

e)



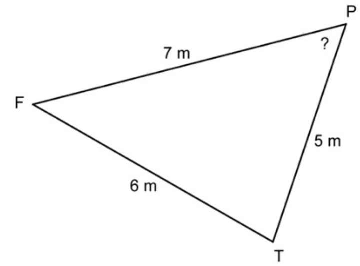
$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{a}{\sin(54)} = \frac{13}{\sin(67)}$$

$$a = \frac{13\sin(54)}{\sin(67)}$$

$$a \simeq 11.43 \text{ cm}$$

f)



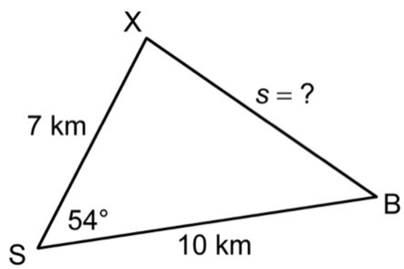
$$\cos P = \frac{6^2 - 7^2 - 5^2}{-2(7)(5)}$$

$$\cos P = \frac{-38}{-70}$$

$$\angle P = \cos^{-1}\left(\frac{38}{70}\right)$$

$$\angle P \simeq 57.12^\circ$$

g)



$$s^2 = 7^2 + 10^2 - 2(7)(10)\cos(54)$$

$$s^2 = 149 - 140\cos(54)$$

$$s^2 = 66.71006468$$

$$s \simeq 8.17 \text{ km}$$